

# High school geometry theorems

Hilbert's axiomatic system.  
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**Theorem 1 (th\_6c\_01.)** *Assuming that  $p \neq q$  and  $q \neq r$  and  $r \neq p$  and lines  $p$  and  $q$  intersect and lines  $q$  and  $r$  intersect and lines  $r$  and  $p$  intersect and  $A \in p$  and  $A \in q$  and  $A \notin r$  and  $B \notin p$  and  $B \in q$  and  $B \in r$  and  $C \in p$  and  $C \notin q$  and  $C \in r$  it holds that  $A \neq B$  and  $B \neq C$  and  $C \neq A$ .*

*Proof:*

1. It holds that  $A = B$  or  $A \neq B$ .
2. Assume that:  $A = B$ .
3. From the facts  $B \notin p$  and  $A = B$  it holds that  $A \notin p$ .
4. From the facts  $A \notin p$  and  $A \in p$  we get contradiction.
5. Assume that:  $A \neq B$ .
6. It holds that  $A = C$  or  $A \neq C$ .
7. Assume that:  $A = C$ .
8. From the facts  $C \notin q$  and  $A = C$  it holds that  $A \notin q$ .
9. From the facts  $A \notin q$  and  $A \in q$  we get contradiction.
10. Assume that:  $A \neq C$ .
11. From the fact  $A \neq C$  it holds that  $C \neq A$ .
12. It holds that  $B = C$  or  $B \neq C$ .
13. Assume that:  $B = C$ .
14. From the facts  $C \in p$  and  $B = C$  it holds that  $B \in p$ .
15. From the facts  $B \notin p$  and  $B \in p$  we get contradiction.
16. Assume that:  $B \neq C$ .
17. The conclusion follows from the facts  $A \neq B$  and  $B \neq C$  and  $C \neq A$ .
18. The conjecture follows in all cases.
19. The conjecture follows in all cases.
20. The conjecture follows in all cases.

QED

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**Theorem 2 (th\_6c\_02.)** *Assuming that  $p \neq q$  and  $q \neq r$  and  $r \neq p$  and lines  $p$  and  $q$  intersect and lines  $q$  and  $r$  intersect and lines  $r$  and  $p$  intersect and  $A \in p$  and  $A \in q$  and  $A \notin r$  and  $B \notin p$  and  $B \in q$  and  $B \in r$  and  $C \in p$  and  $C \notin q$  and  $C \in r$  and  $A \neq B$  and  $B \neq C$  and  $C \neq A$  it holds that  $col(A, B, C)$  or  $\neg col(A, B, C)$ .*

*Proof:*

1. It holds that  $col(A, B, C)$  or  $\neg col(A, B, C)$ .
2. Assume that:  $col(A, B, C)$ .
3. The conclusion follows from the fact  $col(A, B, C)$ .
4. Assume that:  $\neg col(A, B, C)$ .
5. The conclusion follows from the fact  $\neg col(A, B, C)$ .
6. The conjecture follows in all cases.

QED

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**Theorem 3 (th\_6c\_03.)** *Assuming that  $p \neq q$  and  $q \neq r$  and  $r \neq p$  and lines  $p$  and  $q$  intersect and lines  $q$  and  $r$  intersect and lines  $r$  and  $p$  intersect and  $A \in p$  and  $A \in q$  and  $A \notin r$  and  $B \notin p$  and  $B \in q$  and  $B \in r$  and  $C \in p$  and  $C \notin q$  and  $C \in r$  and  $A \neq B$  and  $B \neq C$  and  $C \neq A$  and  $col(A, B, C)$  there exist line  $s$ , such that  $A \in s$  and  $B \in s$  and  $C \in s$ .*

*Proof:*

1. From the fact  $col(A, B, C)$  there exist a line  $s$  where  $A \in s$  and  $B \in s$  and  $C \in s$  (using  $ax.D2$ ).
2. The conclusion follows from the facts  $A \in s$  and  $B \in s$  and  $C \in s$ .

QED

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**Theorem 4 (th\_6c\_04.)** *Assuming that  $p \neq q$  and  $q \neq r$  and  $r \neq p$  and lines  $p$  and  $q$  intersect and lines  $q$  and  $r$  intersect and lines  $r$  and  $p$  intersect and  $A \in p$  and  $A \in q$  and  $A \notin r$  and  $B \notin p$  and  $B \in q$  and  $B \in r$  and  $C \in p$  and  $C \notin q$  and  $C \in r$  and  $A \neq B$  and  $B \neq C$  and  $C \neq A$  and  $col(A, B, C)$  and  $A \in s$  and  $B \in s$  and  $C \in s$  it holds that  $s = p$  and  $s = q$  and  $s = r$ .*

*Proof:*

1. From the facts  $A \neq B$  and  $A \in q$  and  $B \in q$  and  $A \in s$  and  $B \in s$  it holds that  $q = s$  (using  $ax.I2$ ).
2. From the facts  $C \in s$  and  $q = s$  it holds that  $C \in q$ .
3. From the facts  $C \notin q$  and  $C \in q$  we get contradiction.

QED

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**Theorem 5 (th\_6c\_05.)** *Assuming that  $p \neq q$  and  $q \neq r$  and  $r \neq p$  and lines  $p$  and  $q$  intersect and lines  $q$  and  $r$  intersect and lines  $r$  and  $p$  intersect and  $A \in p$  and  $A \in q$  and  $A \notin r$  and  $B \notin p$  and  $B \in q$  and  $B \in r$  and  $C \in p$  and  $C \notin q$  and  $C \in r$  and  $A \neq B$  and  $B \neq C$  and  $C \neq A$  and  $\neg col(A, B, C)$  there exist plane  $\alpha$ , such that  $A \in \alpha$  and  $B \in \alpha$  and  $C \in \alpha$ .*

*Proof:*

1. From the fact  $\neg col(A, B, C)$  there exist a plane  $\alpha$ , where  $A \in \alpha$  and  $B \in \alpha$  and  $C \in \alpha$  (using  $ax.I4a$ ).
2. The conclusion follows from the facts  $A \in \alpha$  and  $B \in \alpha$  and  $C \in \alpha$ .

QED

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**Theorem 6 (th\_6c\_06.)** *Assuming that  $p \neq q$  and  $q \neq r$  and  $r \neq p$  and lines  $p$  and  $q$  intersect and lines  $q$  and  $r$  intersect and lines  $r$  and  $p$  intersect and  $A \in p$  and  $A \in q$  and  $A \notin r$  and  $B \notin p$  and  $B \in q$  and  $B \in r$  and  $C \in p$  and  $C \notin q$  and  $C \in r$  and  $A \neq B$  and  $B \neq C$  and  $C \neq A$  and  $\neg col(A, B, C)$  and  $A \in \alpha$  and  $B \in \alpha$  and  $C \in \alpha$  it holds that  $p \in \alpha$  and  $q \in \alpha$  and  $r \in \alpha$ .*

*Proof:*

1. From the facts  $A \neq B$  and  $A \in q$  and  $B \in q$  and  $A \in \alpha$  and  $B \in \alpha$  it holds that  $q \in \alpha$  (using *ax\_I6*).
2. From the fact  $C \neq A$  it holds that  $A \neq C$ .
3. From the facts  $A \neq C$  and  $A \in p$  and  $C \in p$  and  $A \in \alpha$  and  $C \in \alpha$  it holds that  $p \in \alpha$  (using *ax\_I6*).
4. From the facts  $B \neq C$  and  $B \in r$  and  $C \in r$  and  $B \in \alpha$  and  $C \in \alpha$  it holds that  $r \in \alpha$  (using *ax\_I6*).
5. The conclusion follows from the facts  $p \in \alpha$  and  $q \in \alpha$  and  $r \in \alpha$ .

QED

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**Theorem 7 (th\_6c\_07.)** *Assuming that  $p \neq q$  and  $q \neq r$  and  $r \neq p$  and lines  $p$  and  $q$  intersect and lines  $q$  and  $r$  intersect and lines  $r$  and  $p$  intersect and  $A \in p$  and  $A \in q$  and  $A \notin r$  and  $B \notin p$  and  $B \in q$  and  $B \in r$  and  $C \in p$  and  $C \notin q$  and  $C \in r$  and  $A \neq B$  and  $B \neq C$  and  $C \neq A$  and  $\neg \text{col}(A, B, C)$  and  $A \in \alpha$  and  $B \in \alpha$  and  $C \in \alpha$  it holds that  $p \in \alpha$  and  $q \in \alpha$  and  $r \in \alpha$ .*

*Proof:*

1. From the facts  $A \neq B$  and  $A \in q$  and  $B \in q$  and  $A \in \alpha$  and  $B \in \alpha$  it holds that  $q \in \alpha$  (using *ax\_I6*).
2. From the fact  $C \neq A$  it holds that  $A \neq C$ .
3. From the facts  $A \neq C$  and  $A \in p$  and  $C \in p$  and  $A \in \alpha$  and  $C \in \alpha$  it holds that  $p \in \alpha$  (using *ax\_I6*).
4. From the facts  $B \neq C$  and  $B \in r$  and  $C \in r$  and  $B \in \alpha$  and  $C \in \alpha$  it holds that  $r \in \alpha$  (using *ax\_I6*).
5. The conclusion follows from the facts  $p \in \alpha$  and  $q \in \alpha$  and  $r \in \alpha$ .

QED

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