

High school geometry theorems

Hilbert's axiomatic system.
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Theorem 1 (th_1c_01.) *Assuming that $A \notin p$ there exist point B , point C , such that $B \neq C$ and $B \in p$ and $C \in p$.*

Proof:

1. There exist a point B and a point C where $B \neq C$ and $B \in p$ and $C \in p$ (using ax_I3a).
2. The conclusion follows from the facts $B \neq C$ and $B \in p$ and $C \in p$.

QED

Theorem 2 (th_1c_02.) *Assuming that $A \notin p$ and $B \neq C$ and $B \in p$ and $C \in p$ it holds that $col(A, B, C)$ or $\neg col(A, B, C)$.*

Proof:

1. It holds that $col(A, B, C)$ or $\neg col(A, B, C)$.
2. Assume that: $col(A, B, C)$.
3. The conclusion follows from the fact $col(A, B, C)$.
4. Assume that: $\neg col(A, B, C)$.
5. The conclusion follows from the fact $\neg col(A, B, C)$.
6. The conjecture follows in all cases.

QED

Theorem 3 (th_1c_03.) *Assuming that $A \notin p$ and $B \neq C$ and $B \in p$ and $C \in p$ and $col(A, B, C)$ it holds that $A \in p$.*

Proof:

1. From the fact $col(A, B, C)$ it holds that $col(B, C, A)$ (using ax_sym_col1).
2. From the facts $B \neq C$ and $B \in p$ and $C \in p$ and $A \notin p$ it holds that $\neg col(B, C, A)$ (using ax_D1a).
3. From the facts $\neg col(B, C, A)$ and $col(B, C, A)$ we get contradiction.

QED

Theorem 4 (th_1c_04.) *Assuming that $A \notin p$ and $B \neq C$ and $B \in p$ and $C \in p$ and $\neg col(A, B, C)$ there exist plane α , such that $A \in \alpha$ and $B \in \alpha$ and $C \in \alpha$.*

Proof:

1. From the fact $\neg col(A, B, C)$ there exist a plane α , where $A \in \alpha$ and $B \in \alpha$ and $C \in \alpha$ (using ax_I4a).

2. The conclusion follows from the facts $A \in \alpha$ and $B \in \alpha$ and $C \in \alpha$.

QED

Theorem 5 (th_1c_05.) *Assuming that $A \notin p$ and $B \neq C$ and $B \in p$ and $C \in p$ and $\neg \text{col}(A, B, C)$ and $A \in \alpha$ and $B \in \alpha$ and $C \in \alpha$ it holds that $p \in \alpha$.*

Proof:

1. From the facts $B \neq C$ and $B \in p$ and $C \in p$ and $B \in \alpha$ and $C \in \alpha$ it holds that $p \in \alpha$ (using *ax_I6*).

2. The conclusion follows from the fact $p \in \alpha$.

QED
