

High school geometry theorems

Hilbert's axiomatic system.
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Theorem 1 (th_5n_01.) *Assuming that $\neg col(A, B, C)$ it holds that $A \neq B$ and $B \neq C$ and $C \neq A$.*

Proof:

1. There exist a point D and a point E and a point F and a point G where $\neg cop(D, E, F, G)$ (using ax_I8).
2. It holds that $A = B$ or $A \neq B$.
3. Assume that: $A = B$.
4. It holds that $A = C$ or $A \neq C$.
5. Assume that: $A = C$.
6. From the facts $\neg col(A, B, C)$ and $A = B$ and $A = C$ it holds that $\neg col(A, A, A)$.
7. From the fact $\neg col(A, A, A)$ there exist a plane α , where $A \in \alpha$ and $A \in \alpha$ and $A \in \alpha$ (using ax_I4a).
8. From the facts $A \in \alpha$ and $A \in \alpha$ and $A \in \alpha$ and $A \in \alpha$ it holds that $cop(A, A, A, A)$ (using ax_D3).
9. It holds that $A = D$ or $A \neq D$.
10. Assume that: $A = D$.
11. It holds that $A = E$ or $A \neq E$.
12. Assume that: $A = E$.
13. It holds that $A = F$ or $A \neq F$.
14. Assume that: $A = F$.
15. It holds that $A = G$ or $A \neq G$.
16. Assume that: $A = G$.
17. From the facts $\neg cop(D, E, F, G)$ and $A = B$ and $A = C$ and $A = D$ and $A = E$ and $A = F$ and $A = G$ it holds that $\neg cop(A, A, A, A)$.
18. From the facts $\neg cop(A, A, A, A)$ and $cop(A, A, A, A)$ we get contradiction.
19. Assume that: $A \neq G$.
20. From the fact $A \neq G$ there exist a line p where $A \in p$ and $G \in p$ (using ax_I1).
21. From the facts $A \in p$ and $A \in p$ and $A \in p$ it holds that $col(A, A, A)$ (using ax_D1).

22. From the facts $\neg col(A, B, C)$ and $A = B$ and $A = C$ and $A = D$ and $A = E$ and $A = F$ it holds that $\neg col(A, A, A)$.
23. From the facts $\neg col(A, A, A)$ and $col(A, A, A)$ we get contradiction.
24. The conjecture follows in all cases.
25. Assume that: $A \neq F$.
26. From the fact $A \neq F$ there exist a line p where $A \in p$ and $F \in p$ (using ax_I1).
27. From the facts $\neg col(A, B, C)$ and $A = B$ and $A = C$ and $A = D$ and $A = E$ it holds that $\neg col(A, A, A)$.
28. From the facts $\neg col(A, A, A)$ and $col(A, A, A)$ we get contradiction.
29. The conjecture follows in all cases.
30. Assume that: $A \neq E$.
31. From the fact $A \neq E$ there exist a line p where $A \in p$ and $E \in p$ (using ax_I1).
32. From the facts $\neg col(A, B, C)$ and $A = B$ and $A = C$ and $A = D$ it holds that $\neg col(A, A, A)$.
33. From the facts $\neg col(A, A, A)$ and $col(A, A, A)$ we get contradiction.
34. The conjecture follows in all cases.
35. Assume that: $A \neq D$.
36. From the fact $A \neq D$ there exist a line p where $A \in p$ and $D \in p$ (using ax_I1).
37. From the facts $\neg col(A, B, C)$ and $A = B$ and $A = C$ it holds that $\neg col(A, A, A)$.
38. From the facts $\neg col(A, A, A)$ and $col(A, A, A)$ we get contradiction.
39. The conjecture follows in all cases.
40. Assume that: $A \neq C$.
41. From the facts $\neg col(A, B, C)$ and $A = B$ it holds that $\neg col(A, A, C)$.
42. From the fact $A \neq C$ there exist a line p where $A \in p$ and $C \in p$ (using ax_I1).
43. From the facts $A \in p$ and $A \in p$ and $C \in p$ it holds that $col(A, A, C)$ (using ax_D1).
44. From the facts $\neg col(A, A, C)$ and $col(A, A, C)$ we get contradiction.
45. The conjecture follows in all cases.
46. Assume that: $A \neq B$.
47. From the fact $A \neq B$ there exist a line p where $A \in p$ and $B \in p$ (using ax_I1).
48. It holds that $A = C$ or $A \neq C$.
49. Assume that: $A = C$.
50. From the facts $\neg col(A, B, C)$ and $A = C$ it holds that $\neg col(A, B, A)$.
51. From the facts $A \neq B$ and $A \in p$ and $B \in p$ and $\neg col(A, B, A)$ it holds that $A \notin p$ (using ax_D2a).
52. From the facts $A \notin p$ and $A \in p$ we get contradiction.

53. Assume that: $A \neq C$.
54. From the fact $A \neq C$ it holds that $C \neq A$.
55. It holds that $B = C$ or $B \neq C$.
56. Assume that: $B = C$.
57. From the facts $\neg col(A, B, C)$ and $B = C$ it holds that $\neg col(A, B, B)$.
58. From the facts $A \neq B$ and $A \in p$ and $B \in p$ and $\neg col(A, B, B)$ it holds that $B \notin p$ (using ax_D2a).
59. From the facts $B \notin p$ and $B \in p$ we get contradiction.
60. Assume that: $B \neq C$.
61. The conclusion follows from the facts $A \neq B$ and $B \neq C$ and $C \neq A$.
62. The conjecture follows in all cases.
63. The conjecture follows in all cases.
64. The conjecture follows in all cases.

QED

Theorem 2 (th.5n.1.) *Assuming that $\neg col(A, B, C)$ it holds that $A \neq B$ and $B \neq C$ and $C \neq A$.*

Proof:

1. There exist a point D and a point E and a point F and a point G where $\neg cop(D, E, F, G)$ (using ax_I8).
2. It holds that $A = B$ or $A \neq B$.
3. Assume that: $A = B$.
4. It holds that $A = C$ or $A \neq C$.
5. Assume that: $A = C$.
6. From the facts $\neg col(A, B, C)$ and $A = B$ and $A = C$ it holds that $\neg col(A, A, A)$.
7. From the fact $\neg col(A, A, A)$ there exist a plane α , where $A \in \alpha$ and $A \in \alpha$ and $A \in \alpha$ (using ax_I4a).
8. It holds that $A = D$ or $A \neq D$.
9. Assume that: $A = D$.
10. It holds that $A = E$ or $A \neq E$.
11. Assume that: $A = E$.
12. It holds that $A = F$ or $A \neq F$.
13. Assume that: $A = F$.
14. From the facts $\neg col(A, B, C)$ and $A = B$ and $A = C$ and $A = D$ and $A = E$ and $A = F$ it holds that $\neg col(A, A, A)$.
15. From the facts $\neg cop(D, E, F, G)$ and $A = B$ and $A = C$ and $A = D$ and $A = E$ and $A = F$ it holds that $\neg cop(A, A, A, G)$.
16. From the facts $\neg col(A, A, A)$ and $A \in \alpha$ and $A \in \alpha$ and $A \in \alpha$ and $\neg cop(A, A, A, G)$ it holds that $G \notin \alpha$ (using ax_D4a).
17. It holds that $A = G$ or $A \neq G$.
18. Assume that: $A = G$.

19. From the facts $G \notin \alpha$ and $A = B$ and $A = C$ and $A = D$ and $A = E$ and $A = F$ and $A = G$ it holds that $A \notin \alpha$.
20. From the facts $A \notin \alpha$ and $A \in \alpha$ we get contradiction.
21. Assume that: $A \neq G$.
 22. From the fact $A \neq G$ there exist a line p where $A \in p$ and $G \in p$ (using ax_I1).
 23. From the facts $A \in p$ and $A \in p$ and $A \in p$ it holds that $col(A, A, A)$ (using ax_D1).
 24. From the facts $\neg col(A, B, C)$ and $A = B$ and $A = C$ and $A = D$ and $A = E$ and $A = F$ it holds that $\neg col(A, A, A)$.
 25. From the facts $\neg col(A, A, A)$ and $col(A, A, A)$ we get contradiction.
26. The conjecture follows in all cases.
27. Assume that: $A \neq F$.
 28. From the fact $A \neq F$ there exist a line p where $A \in p$ and $F \in p$ (using ax_I1).
 29. From the facts $\neg col(A, B, C)$ and $A = B$ and $A = C$ and $A = D$ and $A = E$ it holds that $\neg col(A, A, A)$.
 30. From the facts $\neg col(A, A, A)$ and $col(A, A, A)$ we get contradiction.
31. The conjecture follows in all cases.
32. Assume that: $A \neq E$.
 33. From the fact $A \neq E$ there exist a line p where $A \in p$ and $E \in p$ (using ax_I1).
 34. From the facts $\neg col(A, B, C)$ and $A = B$ and $A = C$ and $A = D$ it holds that $\neg col(A, A, A)$.
 35. From the facts $\neg col(A, A, A)$ and $col(A, A, A)$ we get contradiction.
36. The conjecture follows in all cases.
37. Assume that: $A \neq D$.
 38. From the fact $A \neq D$ there exist a line p where $A \in p$ and $D \in p$ (using ax_I1).
 39. From the facts $\neg col(A, B, C)$ and $A = B$ and $A = C$ it holds that $\neg col(A, A, A)$.
 40. From the facts $\neg col(A, A, A)$ and $col(A, A, A)$ we get contradiction.
41. The conjecture follows in all cases.
42. Assume that: $A \neq C$.
 43. From the fact $A \neq C$ there exist a line p where $A \in p$ and $C \in p$ (using ax_I1).
 44. From the facts $A \in p$ and $A \in p$ and $C \in p$ it holds that $col(A, A, C)$ (using ax_D1).
 45. From the facts $\neg col(A, B, C)$ and $A = B$ it holds that $\neg col(A, A, C)$.
 46. From the facts $\neg col(A, A, C)$ and $col(A, A, C)$ we get contradiction.
47. The conjecture follows in all cases.

48. Assume that: $A \neq B$.
49. From the fact $A \neq B$ there exist a line p where $A \in p$ and $B \in p$ (using ax_I1).
50. From the facts $A \in p$ and $B \in p$ and $A \in p$ it holds that $col(A, B, A)$ (using ax_D1).
51. From the facts $A \in p$ and $B \in p$ and $B \in p$ it holds that $col(A, B, B)$ (using ax_D1).
52. It holds that $A = C$ or $A \neq C$.
53. Assume that: $A = C$.
54. From the facts $\neg col(A, B, C)$ and $A = C$ it holds that $\neg col(A, B, A)$.
55. From the facts $\neg col(A, B, A)$ and $col(A, B, A)$ we get contradiction.
56. Assume that: $A \neq C$.
57. From the fact $A \neq C$ it holds that $C \neq A$.
58. It holds that $B = C$ or $B \neq C$.
59. Assume that: $B = C$.
60. From the facts $\neg col(A, B, C)$ and $B = C$ it holds that $\neg col(A, B, B)$.
61. From the facts $\neg col(A, B, B)$ and $col(A, B, B)$ we get contradiction.
62. Assume that: $B \neq C$.
63. The conclusion follows from the facts $A \neq B$ and $B \neq C$ and $C \neq A$.
64. The conjecture follows in all cases.
65. The conjecture follows in all cases.
66. The conjecture follows in all cases.

QED
