

High school geometry theorems

Hilbert's axiomatic system.
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Theorem 1 (th_5c2_01.) *Assuming that $\neg col(A, B, C)$ it holds that $A = B$ or $A \neq B$.*

Theorem 2 (th_5c2_02.) *Assuming that $\neg col(A, B, C)$ and $A = B$ it holds that $B = C$ or $B \neq C$.*

Theorem 3 (th_5c2_03.) *Assuming that $\neg col(A, B, C)$ and $A = B$ and $B = C$ it holds that $col(A, B, C)$.*

Proof:

1. From the facts $\neg col(A, B, C)$ and $A = B$ and $B = C$ it holds that $\neg col(A, A, A)$.
2. From the fact $\neg col(A, A, A)$ there exist a plane α , where $A \in \alpha$ and $A \in \alpha$ and $A \in \alpha$ (using *ax_I4a*).
3. There exist a point D and a point E and a point F and a point G where $\neg cop(D, E, F, G)$ (using *ax_I8*).
4. It holds that $A = D$ or $A \neq D$.
5. Assume that: $A = D$.
6. It holds that $A = E$ or $A \neq E$.
7. Assume that: $A = E$.
8. It holds that $A = F$ or $A \neq F$.
9. Assume that: $A = F$.
10. From the facts $\neg col(A, B, C)$ and $A = B$ and $B = C$ and $A = D$ and $A = E$ and $A = F$ it holds that $\neg col(A, A, A)$.
11. From the facts $\neg cop(D, E, F, G)$ and $A = B$ and $B = C$ and $A = D$ and $A = E$ and $A = F$ it holds that $\neg cop(A, A, A, G)$.
12. From the facts $\neg col(A, A, A)$ and $A \in \alpha$ and $A \in \alpha$ and $A \in \alpha$ and $\neg cop(A, A, A, G)$ it holds that $G \notin \alpha$ (using *ax_D4a*).
13. It holds that $A = G$ or $A \neq G$.
14. Assume that: $A = G$.
15. From the facts $G \notin \alpha$ and $A = B$ and $B = C$ and $A = D$ and $A = E$ and $A = F$ and $A = G$ it holds that $A \notin \alpha$.
16. From the facts $A \notin \alpha$ and $A \in \alpha$ we get contradiction.
17. Assume that: $A \neq G$.
18. From the fact $A \neq G$ there exist a line p where $A \in p$ and $G \in p$ (using *ax_I1*).

19. From the facts $A \in p$ and $A \in p$ and $A \in p$ it holds that $col(A, A, A)$ (using ax_D1).
20. From the facts $\neg col(A, B, C)$ and $A = B$ and $B = C$ and $A = D$ and $A = E$ and $A = F$ it holds that $\neg col(A, A, A)$.
21. From the facts $\neg col(A, A, A)$ and $col(A, A, A)$ we get contradiction.
22. The conjecture follows in all cases.
23. Assume that: $A \neq F$.
24. From the fact $A \neq F$ there exist a line p where $A \in p$ and $F \in p$ (using ax_I1).
25. From the facts $\neg col(A, B, C)$ and $A = B$ and $B = C$ and $A = D$ and $A = E$ it holds that $\neg col(A, A, A)$.
26. From the facts $\neg col(A, A, A)$ and $col(A, A, A)$ we get contradiction.
27. The conjecture follows in all cases.
28. Assume that: $A \neq E$.
29. From the fact $A \neq E$ there exist a line p where $A \in p$ and $E \in p$ (using ax_I1).
30. From the facts $\neg col(A, B, C)$ and $A = B$ and $B = C$ and $A = D$ it holds that $\neg col(A, A, A)$.
31. From the facts $\neg col(A, A, A)$ and $col(A, A, A)$ we get contradiction.
32. The conjecture follows in all cases.
33. Assume that: $A \neq D$.
34. From the fact $A \neq D$ there exist a line p where $A \in p$ and $D \in p$ (using ax_I1).
35. From the facts $\neg col(A, B, C)$ and $A = B$ and $B = C$ it holds that $\neg col(A, A, A)$.
36. From the facts $\neg col(A, A, A)$ and $col(A, A, A)$ we get contradiction.
37. The conjecture follows in all cases.

QED

Theorem 4 (th_5c2_04.) *Assuming that $\neg col(A, B, C)$ and $A = B$ and $B \neq C$ there exist line p , such that $B \in \alpha$ and $C \in \alpha$.*

Proof:

1. From the facts $\neg col(A, B, C)$ and $A = B$ it holds that $\neg col(A, A, C)$.
2. From the fact $\neg col(A, A, C)$ it holds that $\neg col(A, C, A)$ (using ax_sym_ncol1).
3. From the facts $B \neq C$ and $A = B$ it holds that $A \neq C$.
4. From the fact $A \neq C$ there exist a line p where $A \in p$ and $C \in p$ (using ax_I1).
5. From the facts $A \neq C$ and $A \in p$ and $C \in p$ and $\neg col(A, C, A)$ it holds that $A \notin p$ (using ax_D2a).
6. From the facts $A \notin p$ and $A \in p$ we get contradiction.

QED

Theorem 5 (th_5c2_05.) *Assuming that $\neg col(A, B, C)$ and $A = B$ and $B \neq C$ and $B \in \alpha$ and $C \in \alpha$ it holds that $A \in p$.*

Proof:

1. From the facts $\neg col(A, B, C)$ and $A = B$ it holds that $\neg col(A, A, C)$.
2. From the fact $\neg col(A, A, C)$ it holds that $\neg col(A, C, A)$ (using *ax_sym_ncol1*).
3. From the facts $B \neq C$ and $A = B$ it holds that $A \neq C$.
4. From the fact $A \neq C$ there exist a line q where $A \in q$ and $C \in q$ (using *ax_I1*).
5. From the facts $A \neq C$ and $A \in q$ and $C \in q$ and $\neg col(A, C, A)$ it holds that $A \notin q$ (using *ax_D2a*).
6. From the facts $A \notin q$ and $A \in q$ we get contradiction.

QED

Theorem 6 (th_5c2_06.) *Assuming that $\neg col(A, B, C)$ and $A = B$ and $B \neq C$ and $B \in \alpha$ and $C \in \alpha$ and $A \in p$ it holds that $col(A, B, C)$.*

Proof:

1. From the facts $\neg col(A, B, C)$ and $A = B$ it holds that $\neg col(A, A, C)$.
2. From the fact $\neg col(A, A, C)$ it holds that $\neg col(A, C, A)$ (using *ax_sym_ncol1*).
3. From the facts $B \neq C$ and $A = B$ it holds that $A \neq C$.
4. From the fact $A \neq C$ there exist a line q where $A \in q$ and $C \in q$ (using *ax_I1*).
5. From the facts $A \neq C$ and $A \in q$ and $C \in q$ and $\neg col(A, C, A)$ it holds that $A \notin q$ (using *ax_D2a*).
6. From the facts $A \notin q$ and $A \in q$ we get contradiction.

QED

Theorem 7 (th_5c2_07.) *Assuming that $\neg col(A, B, C)$ and $A \neq B$ it holds that $B \neq C$ and $C \neq A$.*

Proof:

1. From the fact $A \neq B$ there exist a line p where $A \in p$ and $B \in p$ (using *ax_I1*).
2. It holds that $A = C$ or $A \neq C$.
3. Assume that: $A = C$.
 4. From the facts $\neg col(A, B, C)$ and $A = C$ it holds that $\neg col(A, B, A)$.
 5. From the facts $A \neq B$ and $A \in p$ and $B \in p$ and $\neg col(A, B, A)$ it holds that $A \notin p$ (using *ax_D2a*).
 6. From the facts $A \notin p$ and $A \in p$ we get contradiction.
7. Assume that: $A \neq C$.
 8. From the fact $A \neq C$ it holds that $C \neq A$.
 9. It holds that $B = C$ or $B \neq C$.
 10. Assume that: $B = C$.
 11. From the facts $\neg col(A, B, C)$ and $B = C$ it holds that $\neg col(A, B, B)$.
 12. From the facts $A \neq B$ and $A \in p$ and $B \in p$ and $\neg col(A, B, B)$ it holds that $B \notin p$ (using *ax_D2a*).
 13. From the facts $B \notin p$ and $B \in p$ we get contradiction.
 14. Assume that: $B \neq C$.
 15. The conclusion follows from the facts $B \neq C$ and $C \neq A$.
16. The conjecture follows in all cases.
17. The conjecture follows in all cases.

QED
