

High school geometry theorems

Hilbert's axiomatic system.
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Theorem 1 (th_5c_01.) *Assuming that $\neg col(A, B, C)$ it holds that $A = B$ or $A \neq B$.*

Theorem 2 (th_5c_02.) *Assuming that $\neg col(A, B, C)$ and $A = B$ it holds that $col(A, B, C)$.*

Proof:

1. From the facts $\neg col(A, B, C)$ and $A = B$ it holds that $\neg col(A, A, C)$.
2. From the fact $\neg col(A, A, C)$ it holds that $\neg col(A, C, A)$ (using *ax_sym_ncol1*).
3. There exist a point D and a point E and a point F and a point G where $\neg cop(D, E, F, G)$ (using *ax_I8*).
4. It holds that $A = C$ or $A \neq C$.
5. Assume that: $A = C$.
 6. From the facts $\neg col(A, B, C)$ and $A = B$ and $A = C$ it holds that $\neg col(A, A, A)$.
 7. From the fact $\neg col(A, A, A)$ there exist a plane α , where $A \in \alpha$ and $A \in \alpha$ and $A \in \alpha$ (using *ax_I4a*).
 8. It holds that $A = D$ or $A \neq D$.
 9. Assume that: $A = D$.
 10. It holds that $A = E$ or $A \neq E$.
 11. Assume that: $A = E$.
 12. It holds that $A = F$ or $A \neq F$.
 13. Assume that: $A = F$.
 14. From the facts $\neg col(A, B, C)$ and $A = B$ and $A = C$ and $A = D$ and $A = E$ and $A = F$ it holds that $\neg col(A, A, A)$.
 15. From the facts $\neg cop(D, E, F, G)$ and $A = B$ and $A = C$ and $A = D$ and $A = E$ and $A = F$ it holds that $\neg cop(A, A, A, G)$.
 16. From the facts $\neg col(A, A, A)$ and $A \in \alpha$ and $A \in \alpha$ and $A \in \alpha$ and $\neg cop(A, A, A, G)$ it holds that $G \notin \alpha$ (using *ax_D4a*).
 17. It holds that $A = G$ or $A \neq G$.
 18. Assume that: $A = G$.
 19. From the facts $G \notin \alpha$ and $A = B$ and $A = C$ and $A = D$ and $A = E$ and $A = F$ and $A = G$ it holds that $A \notin \alpha$.
 20. From the facts $A \notin \alpha$ and $A \in \alpha$ we get contradiction.

21. Assume that: $A \neq G$.
22. From the fact $A \neq G$ there exist a line p where $A \in p$ and $G \in p$ (using ax_I1).
23. From the facts $A \in p$ and $A \in p$ and $A \in p$ it holds that $col(A, A, A)$ (using ax_D1).
24. From the facts $\neg col(A, B, C)$ and $A = B$ and $A = C$ and $A = D$ and $A = E$ and $A = F$ it holds that $\neg col(A, A, A)$.
25. From the facts $\neg col(A, A, A)$ and $col(A, A, A)$ we get contradiction.
26. The conjecture follows in all cases.
27. Assume that: $A \neq F$.
28. From the fact $A \neq F$ there exist a line p where $A \in p$ and $F \in p$ (using ax_I1).
29. From the facts $\neg col(A, B, C)$ and $A = B$ and $A = C$ and $A = D$ and $A = E$ it holds that $\neg col(A, A, A)$.
30. From the facts $\neg col(A, A, A)$ and $col(A, A, A)$ we get contradiction.
31. The conjecture follows in all cases.
32. Assume that: $A \neq E$.
33. From the fact $A \neq E$ there exist a line p where $A \in p$ and $E \in p$ (using ax_I1).
34. From the facts $\neg col(A, B, C)$ and $A = B$ and $A = C$ and $A = D$ it holds that $\neg col(A, A, A)$.
35. From the facts $\neg col(A, A, A)$ and $col(A, A, A)$ we get contradiction.
36. The conjecture follows in all cases.
37. Assume that: $A \neq D$.
38. From the fact $A \neq D$ there exist a line p where $A \in p$ and $D \in p$ (using ax_I1).
39. From the facts $\neg col(A, B, C)$ and $A = B$ and $A = C$ it holds that $\neg col(A, A, A)$.
40. From the facts $\neg col(A, A, A)$ and $col(A, A, A)$ we get contradiction.
41. The conjecture follows in all cases.
42. Assume that: $A \neq C$.
43. From the fact $A \neq C$ there exist a line p where $A \in p$ and $C \in p$ (using ax_I1).
44. From the facts $A \neq C$ and $A \in p$ and $C \in p$ and $\neg col(A, C, A)$ it holds that $A \notin p$ (using ax_D2a).
45. From the facts $A \notin p$ and $A \in p$ we get contradiction.
46. The conjecture follows in all cases.

QED

Theorem 3 (th_5c_03.) *Assuming that $\neg col(A, B, C)$ and $A \neq B$ it holds that $B \neq C$ and $C \neq A$.*

Proof:

1. From the fact $A \neq B$ there exist a line p where $A \in p$ and $B \in p$ (using ax_I1).

2. It holds that $A = C$ or $A \neq C$.
3. Assume that: $A = C$.
 4. From the facts $\neg col(A, B, C)$ and $A = C$ it holds that $\neg col(A, B, A)$.
 5. From the facts $A \neq B$ and $A \in p$ and $B \in p$ and $\neg col(A, B, A)$ it holds that $A \notin p$ (using ax_D2a).
 6. From the facts $A \notin p$ and $A \in p$ we get contradiction.
7. Assume that: $A \neq C$.
 8. From the fact $A \neq C$ it holds that $C \neq A$.
 9. It holds that $B = C$ or $B \neq C$.
 10. Assume that: $B = C$.
 11. From the facts $\neg col(A, B, C)$ and $B = C$ it holds that $\neg col(A, B, B)$.
 12. From the facts $A \neq B$ and $A \in p$ and $B \in p$ and $\neg col(A, B, B)$ it holds that $B \notin p$ (using ax_D2a).
 13. From the facts $B \notin p$ and $B \in p$ we get contradiction.
 14. Assume that: $B \neq C$.
 15. The conclusion follows from the facts $B \neq C$ and $C \neq A$.
16. The conjecture follows in all cases.
17. The conjecture follows in all cases.

QED
