

High school geometry theorems

Hilbert's axiomatic system.
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Theorem 1 (th_5c3_01.) *Assuming that $\neg col(A, B, C)$ and $A = B$ and $B = C$ it holds that $col(A, B, C)$.*

Proof:

1. From the facts $\neg col(A, B, C)$ and $A = B$ and $B = C$ it holds that $\neg col(A, A, A)$.
2. From the fact $\neg col(A, A, A)$ there exist a plane α , where $A \in \alpha$ and $A \in \alpha$ and $A \in \alpha$ (using *ax_I4a*).
3. There exist a point D and a point E and a point F and a point G where $\neg cop(D, E, F, G)$ (using *ax_I8*).
4. It holds that $A = D$ or $A \neq D$.
5. Assume that: $A = D$.
6. It holds that $A = E$ or $A \neq E$.
7. Assume that: $A = E$.
8. It holds that $A = F$ or $A \neq F$.
9. Assume that: $A = F$.
10. From the facts $\neg col(A, B, C)$ and $A = B$ and $B = C$ and $A = D$ and $A = E$ and $A = F$ it holds that $\neg col(A, A, A)$.
11. From the facts $\neg cop(D, E, F, G)$ and $A = B$ and $B = C$ and $A = D$ and $A = E$ and $A = F$ it holds that $\neg cop(A, A, A, G)$.
12. From the facts $\neg col(A, A, A)$ and $A \in \alpha$ and $A \in \alpha$ and $A \in \alpha$ and $\neg cop(A, A, A, G)$ it holds that $G \notin \alpha$ (using *ax_D4a*).
13. It holds that $A = G$ or $A \neq G$.
14. Assume that: $A = G$.
15. From the facts $G \notin \alpha$ and $A = B$ and $B = C$ and $A = D$ and $A = E$ and $A = F$ and $A = G$ it holds that $A \notin \alpha$.
16. From the facts $A \notin \alpha$ and $A \in \alpha$ we get contradiction.
17. Assume that: $A \neq G$.
18. From the fact $A \neq G$ there exist a line p where $A \in p$ and $G \in p$ (using *ax_I1*).
19. From the facts $A \in p$ and $A \in p$ and $A \in p$ it holds that $col(A, A, A)$ (using *ax_D1*).
20. From the facts $\neg col(A, B, C)$ and $A = B$ and $B = C$ and $A = D$ and $A = E$ and $A = F$ it holds that $\neg col(A, A, A)$.
21. From the facts $\neg col(A, A, A)$ and $col(A, A, A)$ we get contradiction.
22. The conjecture follows in all cases.

23. Assume that: $A \neq F$.
24. From the fact $A \neq F$ there exist a line p where $A \in p$ and $F \in p$ (using ax_I1).
25. From the facts $\neg col(A, B, C)$ and $A = B$ and $B = C$ and $A = D$ and $A = E$ it holds that $\neg col(A, A, A)$.
26. From the facts $\neg col(A, A, A)$ and $col(A, A, A)$ we get contradiction.
27. The conjecture follows in all cases.
28. Assume that: $A \neq E$.
29. From the fact $A \neq E$ there exist a line p where $A \in p$ and $E \in p$ (using ax_I1).
30. From the facts $\neg col(A, B, C)$ and $A = B$ and $B = C$ and $A = D$ it holds that $\neg col(A, A, A)$.
31. From the facts $\neg col(A, A, A)$ and $col(A, A, A)$ we get contradiction.
32. The conjecture follows in all cases.
33. Assume that: $A \neq D$.
34. From the fact $A \neq D$ there exist a line p where $A \in p$ and $D \in p$ (using ax_I1).
35. From the facts $\neg col(A, B, C)$ and $A = B$ and $B = C$ it holds that $\neg col(A, A, A)$.
36. From the facts $\neg col(A, A, A)$ and $col(A, A, A)$ we get contradiction.
37. The conjecture follows in all cases.

QED

Theorem 2 (th_5c3_02.) *Assuming that $\neg col(A, B, C)$ and $A = B$ and $B \neq C$ there exist line p , such that $B \in p$ and $C \in p$.*

Proof:

1. From the facts $B \neq C$ and $A = B$ it holds that $A \neq C$.
2. From the fact $A \neq C$ there exist a line p where $A \in p$ and $C \in p$ (using ax_I1).
3. From the facts $A \in p$ and $A = B$ it holds that $B \in p$.
4. The conclusion follows from the facts $B \in p$ and $C \in p$.

QED

Theorem 3 (th_5c3_03.) *Assuming that $\neg col(A, B, C)$ and $A = B$ and $B \neq C$ and $B \in s$ and $C \in s$ it holds that $A \in s$.*

Theorem 4 (th_5c3_04.) *Assuming that $\neg col(A, B, C)$ and $A = B$ and $B \neq C$ and $B \in p$ and $C \in p$ and $A \in p$ it holds that $col(A, B, C)$.*

Proof:

1. From the facts $B \in p$ and $A = B$ it holds that $A \in p$.
2. From the facts $A \in p$ and $A \in p$ and $C \in p$ it holds that $col(A, A, C)$ (using ax_D1).
3. From the facts $\neg col(A, B, C)$ and $A = B$ it holds that $\neg col(A, A, C)$.
4. From the facts $\neg col(A, A, C)$ and $col(A, A, C)$ we get contradiction.
5. From the facts $col(A, A, C)$ and $A = B$ it holds that $col(A, B, C)$.
6. The conclusion follows from the fact $col(A, B, C)$.

QED

Theorem 5 (th_5c3_05.) *Assuming that $\neg col(A, B, C)$ and $A \neq B$ there exist line p , such that $A \in p$ and $B \in p$.*

Proof:

1. From the fact $A \neq B$ there exist a line p where $A \in p$ and $B \in p$ (using ax_I1).
2. The conclusion follows from the facts $A \in p$ and $B \in p$.

QED

Theorem 6 (th_5c3_06.) *Assuming that $\neg col(A, B, C)$ and $A \neq B$ and $A \in s$ and $B \in s$ and $B = C$ it holds that $C \in s$.*

Theorem 7 (th_5c3_07.) *Assuming that $\neg col(A, B, C)$ and $A \neq B$ and $A \in p$ and $B \in p$ and $B = C$ and $C \in p$ it holds that $col(A, B, C)$.*

Proof:

1. From the facts $\neg col(A, B, C)$ and $B = C$ it holds that $\neg col(A, B, B)$.
2. From the facts $A \neq B$ and $A \in p$ and $B \in p$ and $\neg col(A, B, B)$ it holds that $B \notin p$ (using ax_D2a).
3. From the facts $B \notin p$ and $B \in p$ we get contradiction.

QED

Theorem 8 (th_5c3_08.) *Assuming that $\neg col(A, B, C)$ and $A \neq B$ and $A \in p$ and $B \in p$ and $B \neq C$ it holds that $C \neq A$.*

Proof:

1. From the facts $A \neq B$ and $A \in p$ and $B \in p$ and $\neg col(A, B, C)$ it holds that $C \notin p$ (using ax_D2a).
2. It holds that $A = C$ or $A \neq C$.
3. Assume that: $A = C$.
 4. From the facts $C \notin p$ and $A = C$ it holds that $A \notin p$.
 5. From the facts $A \notin p$ and $A \in p$ we get contradiction.
6. Assume that: $A \neq C$.
 7. From the fact $A \neq C$ it holds that $C \neq A$.
 8. The conclusion follows from the fact $C \neq A$.
9. The conjecture follows in all cases.

QED
