

High school geometry theorems

Hilbert's axiomatic system.
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Theorem 1 (th_5_1.) *Assuming that $\neg col(A, B, C)$ it holds that $A \neq B$ and $B \neq C$ and $C \neq A$.*

Proof:

1. There exist a point D and a point E and a point F and a point G where $\neg cop(D, E, F, G)$ (using *ax_I8*).
2. It holds that $A = B$ or $A \neq B$.
3. Assume that: $A = B$.
4. It holds that $A = C$ or $A \neq C$.
5. Assume that: $A = C$.
6. From the facts $\neg col(A, B, C)$ and $A = B$ and $A = C$ it holds that $\neg col(A, A, A)$.
7. From the fact $\neg col(A, A, A)$ there exist a plane α , where $A \in \alpha$ and $A \in \alpha$ and $A \in \alpha$ (using *ax_I4a*).
8. It holds that $A = D$ or $A \neq D$.
9. Assume that: $A = D$.
10. It holds that $A = E$ or $A \neq E$.
11. Assume that: $A = E$.
12. It holds that $A = F$ or $A \neq F$.
13. Assume that: $A = F$.
14. From the facts $\neg col(A, B, C)$ and $A = B$ and $A = C$ and $A = D$ and $A = E$ and $A = F$ it holds that $\neg col(A, A, A)$.
15. From the facts $\neg cop(D, E, F, G)$ and $A = B$ and $A = C$ and $A = D$ and $A = E$ and $A = F$ it holds that $\neg cop(A, A, A, G)$.
16. From the facts $\neg col(A, A, A)$ and $A \in \alpha$ and $A \in \alpha$ and $A \in \alpha$ and $\neg cop(A, A, A, G)$ it holds that $G \notin \alpha$ (using *ax_D4a*).
17. It holds that $A = G$ or $A \neq G$.
18. Assume that: $A = G$.
19. From the facts $G \notin \alpha$ and $A = B$ and $A = C$ and $A = D$ and $A = E$ and $A = F$ and $A = G$ it holds that $A \notin \alpha$.
20. From the facts $A \notin \alpha$ and $A \in \alpha$ we get contradiction.

21. Assume that: $A \neq G$.
22. From the fact $A \neq G$ there exist a line p where $A \in p$ and $G \in p$ (using ax_I1).
23. From the facts $A \in p$ and $A \in p$ and $A \in p$ it holds that $col(A, A, A)$ (using ax_D1).
24. From the facts $\neg col(A, B, C)$ and $A = B$ and $A = C$ and $A = D$ and $A = E$ and $A = F$ it holds that $\neg col(A, A, A)$.
25. From the facts $\neg col(A, A, A)$ and $col(A, A, A)$ we get contradiction.
26. The conjecture follows in all cases.
27. Assume that: $A \neq F$.
28. From the fact $A \neq F$ there exist a line p where $A \in p$ and $F \in p$ (using ax_I1).
29. From the facts $\neg col(A, B, C)$ and $A = B$ and $A = C$ and $A = D$ and $A = E$ it holds that $\neg col(A, A, A)$.
30. From the facts $\neg col(A, A, A)$ and $col(A, A, A)$ we get contradiction.
31. The conjecture follows in all cases.
32. Assume that: $A \neq E$.
33. From the fact $A \neq E$ there exist a line p where $A \in p$ and $E \in p$ (using ax_I1).
34. From the facts $\neg col(A, B, C)$ and $A = B$ and $A = C$ and $A = D$ it holds that $\neg col(A, A, A)$.
35. From the facts $\neg col(A, A, A)$ and $col(A, A, A)$ we get contradiction.
36. The conjecture follows in all cases.
37. Assume that: $A \neq D$.
38. From the fact $A \neq D$ there exist a line p where $A \in p$ and $D \in p$ (using ax_I1).
39. From the facts $\neg col(A, B, C)$ and $A = B$ and $A = C$ it holds that $\neg col(A, A, A)$.
40. From the facts $\neg col(A, A, A)$ and $col(A, A, A)$ we get contradiction.
41. The conjecture follows in all cases.
42. Assume that: $A \neq C$.
43. From the fact $A \neq C$ there exist a line p where $A \in p$ and $C \in p$ (using ax_I1).
44. From the facts $A \in p$ and $A \in p$ and $C \in p$ it holds that $col(A, A, C)$ (using ax_D1).
45. From the facts $\neg col(A, B, C)$ and $A = B$ it holds that $\neg col(A, A, C)$.
46. From the facts $\neg col(A, A, C)$ and $col(A, A, C)$ we get contradiction.
47. The conjecture follows in all cases.
48. Assume that: $A \neq B$.
49. From the fact $A \neq B$ there exist a line p where $A \in p$ and $B \in p$ (using ax_I1).
50. From the facts $A \in p$ and $B \in p$ and $A \in p$ it holds that $col(A, B, A)$ (using ax_D1).

51. From the facts $A \in p$ and $B \in p$ and $B \in p$ it holds that $col(A, B, B)$ (using ax_D1).
 52. It holds that $A = C$ or $A \neq C$.
 53. Assume that: $A = C$.
 54. From the facts $\neg col(A, B, C)$ and $A = C$ it holds that $\neg col(A, B, A)$.
 55. From the facts $\neg col(A, B, A)$ and $col(A, B, A)$ we get contradiction.
 56. Assume that: $A \neq C$.
 57. From the fact $A \neq C$ it holds that $C \neq A$.
 58. It holds that $B = C$ or $B \neq C$.
 59. Assume that: $B = C$.
 60. From the facts $\neg col(A, B, C)$ and $B = C$ it holds that $\neg col(A, B, B)$.
 61. From the facts $\neg col(A, B, B)$ and $col(A, B, B)$ we get contradiction.
 62. Assume that: $B \neq C$.
 63. The conclusion follows from the facts $A \neq B$ and $B \neq C$ and $C \neq A$.
 64. The conjecture follows in all cases.
 65. The conjecture follows in all cases.
 66. The conjecture follows in all cases.
- QED
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