

# High school geometry theorems

Hilbert's axiomatic system.  
Formalized by: Sana Stojanovic Djurdjevic  
Proved by: ArgoGeoChecker.

30.03.2018.

**Theorem 1 (th\_5c3\_01.)** *Assuming that  $A = B$  and  $B = C$  it holds that  $col(A, B, C)$ .*

*Proof:*

1. There exist a point  $D$  and a point  $E$  and a point  $F$  where  $\neg col(D, E, F)$  (using  $ax_I3b$ ).
2. There exist a point  $G$  and a point  $I$  and a point  $J$  and a point  $K$  where  $\neg cop(G, I, J, K)$  (using  $ax_I8$ ).
3. It holds that  $A = D$  or  $A \neq D$ .
4. Assume that:  $A = D$ .
5. It holds that  $A = E$  or  $A \neq E$ .
6. Assume that:  $A = E$ .
7. It holds that  $A = F$  or  $A \neq F$ .
8. Assume that:  $A = F$ .
9. From the facts  $\neg col(D, E, F)$  and  $A = B$  and  $B = C$  and  $A = D$  and  $A = E$  and  $A = F$  it holds that  $\neg col(A, A, A)$ .
10. From the fact  $\neg col(A, A, A)$  there exist a plane  $\alpha$ , where  $A \in \alpha$  and  $A \in \alpha$  and  $A \in \alpha$  (using  $ax_I4a$ ).
11. It holds that  $A = G$  or  $A \neq G$ .
12. Assume that:  $A = G$ .
13. It holds that  $A = I$  or  $A \neq I$ .
14. Assume that:  $A = I$ .
15. It holds that  $A = J$  or  $A \neq J$ .
16. Assume that:  $A = J$ .
17. From the facts  $\neg col(D, E, F)$  and  $A = B$  and  $B = C$  and  $A = D$  and  $A = E$  and  $A = F$  and  $A = G$  and  $A = I$  and  $A = J$  it holds that  $\neg col(A, A, A)$ .
18. From the facts  $\neg cop(G, I, J, K)$  and  $A = B$  and  $B = C$  and  $A = D$  and  $A = E$  and  $A = F$  and  $A = G$  and  $A = I$  and  $A = J$  it holds that  $\neg cop(A, A, A, K)$ .
19. From the facts  $\neg col(A, A, A)$  and  $A \in \alpha$  and  $A \in \alpha$  and  $A \in \alpha$  and  $\neg cop(A, A, A, K)$  it holds that  $K \notin \alpha$  (using  $ax_D4a$ ).
20. It holds that  $A = K$  or  $A \neq K$ .
21. Assume that:  $A = K$ .

22. From the facts  $K \notin \alpha$  and  $A = B$  and  $B = C$  and  $A = D$  and  $A = E$  and  $A = F$  and  $A = G$  and  $A = I$  and  $A = J$  and  $A = K$  it holds that  $A \notin \alpha$ .
23. From the facts  $A \notin \alpha$  and  $A \in \alpha$  we get contradiction.
24. Assume that:  $A \neq K$ .
25. From the fact  $A \neq K$  there exist a line  $p$  where  $A \in p$  and  $K \in p$  (using  $ax\_I1$ ).
26. From the facts  $A \in p$  and  $A \in p$  and  $A \in p$  it holds that  $col(A, A, A)$  (using  $ax\_D1$ ).
27. From the facts  $\neg col(D, E, F)$  and  $A = B$  and  $B = C$  and  $A = D$  and  $A = E$  and  $A = F$  and  $A = G$  and  $A = I$  and  $A = J$  it holds that  $\neg col(A, A, A)$ .
28. From the facts  $\neg col(A, A, A)$  and  $col(A, A, A)$  we get contradiction.
29. The conjecture follows in all cases.
30. Assume that:  $A \neq J$ .
31. From the fact  $A \neq J$  there exist a line  $p$  where  $A \in p$  and  $J \in p$  (using  $ax\_I1$ ).
32. From the facts  $\neg col(D, E, F)$  and  $A = B$  and  $B = C$  and  $A = D$  and  $A = E$  and  $A = F$  and  $A = G$  and  $A = I$  it holds that  $\neg col(A, A, A)$ .
33. From the facts  $\neg col(A, A, A)$  and  $col(A, A, A)$  we get contradiction.
34. The conjecture follows in all cases.
35. Assume that:  $A \neq I$ .
36. From the fact  $A \neq I$  there exist a line  $p$  where  $A \in p$  and  $I \in p$  (using  $ax\_I1$ ).
37. From the facts  $\neg col(D, E, F)$  and  $A = B$  and  $B = C$  and  $A = D$  and  $A = E$  and  $A = F$  and  $A = G$  it holds that  $\neg col(A, A, A)$ .
38. From the facts  $\neg col(A, A, A)$  and  $col(A, A, A)$  we get contradiction.
39. The conjecture follows in all cases.
40. Assume that:  $A \neq G$ .
41. From the fact  $A \neq G$  there exist a line  $p$  where  $A \in p$  and  $G \in p$  (using  $ax\_I1$ ).
42. From the facts  $\neg col(D, E, F)$  and  $A = B$  and  $B = C$  and  $A = D$  and  $A = E$  and  $A = F$  it holds that  $\neg col(A, A, A)$ .
43. From the facts  $\neg col(A, A, A)$  and  $col(A, A, A)$  we get contradiction.
44. The conjecture follows in all cases.
45. Assume that:  $A \neq F$ .
46. From the fact  $A \neq F$  there exist a line  $p$  where  $A \in p$  and  $F \in p$  (using  $ax\_I1$ ).
47. From the facts  $col(A, A, A)$  and  $A = B$  and  $A = C$  it holds that  $col(A, B, C)$ .

48. The conclusion follows from the fact  $col(A, B, C)$ .
49. The conjecture follows in all cases.
50. Assume that:  $A \neq E$ .
51. From the fact  $A \neq E$  there exist a line  $p$  where  $A \in p$  and  $E \in p$  (using  $ax\_I1$ ).
52. From the facts  $col(A, A, A)$  and  $A = B$  and  $A = C$  it holds that  $col(A, B, C)$ .
53. The conclusion follows from the fact  $col(A, B, C)$ .
54. The conjecture follows in all cases.
55. Assume that:  $A \neq D$ .
56. From the fact  $A \neq D$  there exist a line  $p$  where  $A \in p$  and  $D \in p$  (using  $ax\_I1$ ).
57. From the facts  $col(A, A, A)$  and  $A = B$  and  $A = C$  it holds that  $col(A, B, C)$ .
58. The conclusion follows from the fact  $col(A, B, C)$ .
59. The conjecture follows in all cases.

QED

---

**Theorem 2 (th\_5c3\_02.)** *Assuming that  $\neg col(A, B, C)$  and  $A = B$  and  $B \neq C$  there exist line  $p$ , such that  $B \in p$  and  $C \in p$ .*

*Proof:*

1. From the facts  $B \neq C$  and  $A = B$  it holds that  $A \neq C$ .
2. From the fact  $A \neq C$  there exist a line  $p$  where  $A \in p$  and  $C \in p$  (using  $ax\_I1$ ).
3. From the facts  $A \in p$  and  $A = B$  it holds that  $B \in p$ .
4. The conclusion follows from the facts  $B \in p$  and  $C \in p$ .

QED

---

**Theorem 3 (th\_5c3\_03.)** *Assuming that  $A = B$  and  $B \neq C$  and  $B \in p$  and  $C \in p$  it holds that  $col(A, B, C)$ .*

*Proof:*

1. From the facts  $B \in p$  and  $A = B$  it holds that  $A \in p$ .
2. From the facts  $A \in p$  and  $A \in p$  and  $C \in p$  it holds that  $col(A, A, C)$  (using  $ax\_D1$ ).
3. From the facts  $col(A, A, C)$  and  $A = B$  it holds that  $col(A, B, C)$ .
4. The conclusion follows from the fact  $col(A, B, C)$ .

QED

---

**Theorem 4 (th\_5c3\_04.)** *Assuming that  $\neg col(A, B, C)$  and  $A \neq B$  there exist line  $p$ , such that  $A \in p$  and  $B \in p$ .*

*Proof:*

1. From the fact  $A \neq B$  there exist a line  $p$  where  $A \in p$  and  $B \in p$  (using  $ax\_I1$ ).
2. The conclusion follows from the facts  $A \in p$  and  $B \in p$ .

QED

---

**Theorem 5 (th\_5c3\_05.)** *Assuming that  $A \neq B$  and  $A \in p$  and  $B \in p$  and  $B = C$  it holds that  $col(A, B, C)$ .*

*Proof:*

1. From the facts  $A \in p$  and  $B \in p$  and  $B \in p$  it holds that  $col(A, B, B)$  (using  $ax\_D1$ ).
2. From the facts  $col(A, B, B)$  and  $B = C$  it holds that  $col(A, B, C)$ .
3. The conclusion follows from the fact  $col(A, B, C)$ .

QED

---

**Theorem 6 (th\_5c3\_06.)** *Assuming that  $\neg col(A, B, C)$  and  $A \neq B$  and  $A \in p$  and  $B \in p$  and  $B \neq C$  it holds that  $C \neq A$ .*

*Proof:*

1. From the facts  $A \neq B$  and  $A \in p$  and  $B \in p$  and  $\neg col(A, B, C)$  it holds that  $C \notin p$  (using  $ax\_D2a$ ).
2. It holds that  $A = C$  or  $A \neq C$ .
3. Assume that:  $A = C$ .
  4. From the facts  $C \notin p$  and  $A = C$  it holds that  $A \notin p$ .
  5. From the facts  $A \notin p$  and  $A \in p$  we get contradiction.
6. Assume that:  $A \neq C$ .
  7. From the fact  $A \neq C$  it holds that  $C \neq A$ .
  8. The conclusion follows from the fact  $C \neq A$ .
9. The conjecture follows in all cases.

QED

---

**Theorem 7 (th\_5c3\_07.)** *Assuming that  $\neg col(A, B, C)$  and  $A \neq B$  and  $A \in p$  and  $B \in p$  and  $B = C$  and  $C \in p$  it holds that  $col(A, B, C)$ .*

*Proof:*

1. From the facts  $\neg col(A, B, C)$  and  $B = C$  it holds that  $\neg col(A, B, B)$ .
2. From the facts  $A \neq B$  and  $A \in p$  and  $B \in p$  and  $\neg col(A, B, B)$  it holds that  $B \notin p$  (using  $ax\_D2a$ ).
3. From the facts  $B \notin p$  and  $B \in p$  we get contradiction.

QED

---

**Theorem 8 (th\_5c3\_08.)** *Assuming that  $\neg col(A, B, C)$  and  $A \neq B$  and  $A \in p$  and  $B \in p$  and  $B \neq C$  it holds that  $C \neq A$ .*

*Proof:*

1. From the facts  $A \neq B$  and  $A \in p$  and  $B \in p$  and  $\neg col(A, B, C)$  it holds that  $C \notin p$  (using  $ax\_D2a$ ).
2. It holds that  $A = C$  or  $A \neq C$ .
3. Assume that:  $A = C$ .
  4. From the facts  $C \notin p$  and  $A = C$  it holds that  $A \notin p$ .
  5. From the facts  $A \notin p$  and  $A \in p$  we get contradiction.
6. Assume that:  $A \neq C$ .
  7. From the fact  $A \neq C$  it holds that  $C \neq A$ .
  8. The conclusion follows from the fact  $C \neq A$ .
9. The conjecture follows in all cases.

QED

---