

High school geometry theorems

Hilbert's axiomatic system.
Formalized by: Sana Stojanovic Djurdjevic
Proved by: ArgoGeoChecker.

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Theorem 1 (th_5c_01.) *Assuming that $A = B$ it holds that $col(A, B, C)$.*

Proof:

1. There exist a point D and a point E and a point F where $\neg col(D, E, F)$ (using *ax_I3b*).
2. There exist a point G and a point I and a point J and a point K where $\neg cop(G, I, J, K)$ (using *ax_I8*).
3. It holds that $A = C$ or $A \neq C$.
4. Assume that: $A = C$.
5. It holds that $A = D$ or $A \neq D$.
6. Assume that: $A = D$.
7. It holds that $A = E$ or $A \neq E$.
8. Assume that: $A = E$.
9. It holds that $A = F$ or $A \neq F$.
10. Assume that: $A = F$.
11. From the facts $\neg col(D, E, F)$ and $A = B$ and $A = C$ and $A = D$ and $A = E$ and $A = F$ it holds that $\neg col(A, A, A)$.
12. From the fact $\neg col(A, A, A)$ there exist a plane α , where $A \in \alpha$ and $A \in \alpha$ and $A \in \alpha$ (using *ax_I4a*).
13. It holds that $A = G$ or $A \neq G$.
14. Assume that: $A = G$.
15. It holds that $A = I$ or $A \neq I$.
16. Assume that: $A = I$.
17. It holds that $A = J$ or $A \neq J$.
18. Assume that: $A = J$.
19. From the facts $\neg col(D, E, F)$ and $A = B$ and $A = C$ and $A = D$ and $A = E$ and $A = F$ and $A = G$ and $A = I$ and $A = J$ it holds that $\neg col(A, A, A)$.
20. From the facts $\neg cop(G, I, J, K)$ and $A = B$ and $A = C$ and $A = D$ and $A = E$ and $A = F$ and $A = G$ and $A = I$ and $A = J$ it holds that $\neg cop(A, A, A, K)$.
21. From the facts $\neg col(A, A, A)$ and $A \in \alpha$ and $A \in \alpha$ and $A \in \alpha$ and $\neg cop(A, A, A, K)$ it holds that $K \notin \alpha$ (using *ax_D4a*).

22. It holds that $A = K$ or $A \neq K$.
23. Assume that: $A = K$.
 24. From the facts $K \notin \alpha$ and $A = B$ and $A = C$ and $A = D$ and $A = E$ and $A = F$ and $A = G$ and $A = I$ and $A = J$ and $A = K$ it holds that $A \notin \alpha$.
 25. From the facts $A \notin \alpha$ and $A \in \alpha$ we get contradiction.
26. Assume that: $A \neq K$.
 27. From the fact $A \neq K$ there exist a line p where $A \in p$ and $K \in p$ (using *ax_I1*).
 28. From the facts $A \in p$ and $A \in p$ and $A \in p$ it holds that $col(A, A, A)$ (using *ax_D1*).
 29. From the facts $\neg col(D, E, F)$ and $A = B$ and $A = C$ and $A = D$ and $A = E$ and $A = F$ and $A = G$ and $A = I$ and $A = J$ it holds that $\neg col(A, A, A)$.
 30. From the facts $\neg col(A, A, A)$ and $col(A, A, A)$ we get contradiction.
31. The conjecture follows in all cases.
32. Assume that: $A \neq J$.
 33. From the fact $A \neq J$ there exist a line p where $A \in p$ and $J \in p$ (using *ax_I1*).
 34. From the facts $\neg col(D, E, F)$ and $A = B$ and $A = C$ and $A = D$ and $A = E$ and $A = F$ and $A = G$ and $A = I$ it holds that $\neg col(A, A, A)$.
 35. From the facts $\neg col(A, A, A)$ and $col(A, A, A)$ we get contradiction.
36. The conjecture follows in all cases.
37. Assume that: $A \neq I$.
 38. From the fact $A \neq I$ there exist a line p where $A \in p$ and $I \in p$ (using *ax_I1*).
 39. From the facts $\neg col(D, E, F)$ and $A = B$ and $A = C$ and $A = D$ and $A = E$ and $A = F$ and $A = G$ it holds that $\neg col(A, A, A)$.
 40. From the facts $\neg col(A, A, A)$ and $col(A, A, A)$ we get contradiction.
41. The conjecture follows in all cases.
42. Assume that: $A \neq G$.
 43. From the fact $A \neq G$ there exist a line p where $A \in p$ and $G \in p$ (using *ax_I1*).
 44. From the facts $\neg col(D, E, F)$ and $A = B$ and $A = C$ and $A = D$ and $A = E$ and $A = F$ it holds that $\neg col(A, A, A)$.
 45. From the facts $\neg col(A, A, A)$ and $col(A, A, A)$ we get contradiction.
46. The conjecture follows in all cases.
47. Assume that: $A \neq F$.

48. From the fact $A \neq F$ there exist a line p where $A \in p$ and $F \in p$ (using ax_I1).
 49. From the facts $col(A, A, A)$ and $A = B$ and $A = C$ it holds that $col(A, B, C)$.
 50. The conclusion follows from the fact $col(A, B, C)$.
 51. The conjecture follows in all cases.
 52. Assume that: $A \neq E$.
 53. From the fact $A \neq E$ there exist a line p where $A \in p$ and $E \in p$ (using ax_I1).
 54. From the facts $col(A, A, A)$ and $A = B$ and $A = C$ it holds that $col(A, B, C)$.
 55. The conclusion follows from the fact $col(A, B, C)$.
 56. The conjecture follows in all cases.
 57. Assume that: $A \neq D$.
 58. From the fact $A \neq D$ there exist a line p where $A \in p$ and $D \in p$ (using ax_I1).
 59. From the facts $col(A, A, A)$ and $A = B$ and $A = C$ it holds that $col(A, B, C)$.
 60. The conclusion follows from the fact $col(A, B, C)$.
 61. The conjecture follows in all cases.
 62. Assume that: $A \neq C$.
 63. From the fact $A \neq C$ there exist a line p where $A \in p$ and $C \in p$ (using ax_I1).
 64. From the facts $A \in p$ and $A \in p$ and $C \in p$ it holds that $col(A, A, C)$ (using ax_D1).
 65. From the facts $col(A, A, C)$ and $A = B$ it holds that $col(A, B, C)$.
 66. The conclusion follows from the fact $col(A, B, C)$.
 67. The conjecture follows in all cases.
- QED

Theorem 2 (th_5c_02.) *Assuming that $\neg col(A, B, C)$ and $A \neq B$ it holds that $B \neq C$ and $C \neq A$.*

Proof:

1. From the fact $A \neq B$ there exist a line p where $A \in p$ and $B \in p$ (using ax_I1).
2. It holds that $A = C$ or $A \neq C$.
3. Assume that: $A = C$.
4. From the facts $\neg col(A, B, C)$ and $A = C$ it holds that $\neg col(A, B, A)$.
5. From the facts $A \neq B$ and $A \in p$ and $B \in p$ and $\neg col(A, B, A)$ it holds that $A \notin p$ (using ax_D2a).
6. From the facts $A \notin p$ and $A \in p$ we get contradiction.
7. Assume that: $A \neq C$.
8. From the fact $A \neq C$ it holds that $C \neq A$.
9. It holds that $B = C$ or $B \neq C$.
10. Assume that: $B = C$.
11. From the facts $\neg col(A, B, C)$ and $B = C$ it holds that $\neg col(A, B, B)$.

12. From the facts $A \neq B$ and $A \in p$ and $B \in p$ and $\neg col(A, B, B)$ it holds that $B \notin p$ (using *ax_D2a*).
 13. From the facts $B \notin p$ and $B \in p$ we get contradiction.
 14. Assume that: $B \neq C$.
 15. The conclusion follows from the facts $B \neq C$ and $C \neq A$.
 16. The conjecture follows in all cases.
 17. The conjecture follows in all cases.
- QED
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