Examples of proofs in axiomatic system E

Formal system E for Euclid's Elements.
Formalized by: Sana Stojanovic Djurdjevic
Proved by: ArgoAvigadChecker.

07.01.2017.

Theorem 1 (th_prop1_01.) *Assuming that* $A \neq B$ *there exist circle* α*, such that* A *is the center of* α *and* B *is on circle* α*.*

Proof:

1. From the fact $A \neq B$ there exist a circle α, where A is the center of α and B is on circle α (using $ax\_lines\_and\_circles2$).
2. The conclusion follows from the facts A is the center of α and B is on circle α.

QED

Theorem 2 (th_prop1_02.) *Assuming that* $A \neq B$ *and* A *is the center of* α *and* B *is on circle* α *there exist circle* β*, such that* B *is the center of* β *and* A *is on circle* β*.*

Proof:

1. From the fact $A \neq B$ it holds that $B \neq A$.
2. From the fact $B \neq A$ there exist a circle β, where B is the center of β and A is on circle β (using $ax\_lines\_and\_circles2$).
3. The conclusion follows from the facts B is the center of β and A is on circle β.

QED

Theorem 3 (th_prop1_03.) *Assuming that* $A \neq B$ *and* A *is the center of* α *and* B *is on circle* α *and* B *is the center of* β *and* A *is on circle* β *there exist point* C*, such that* C *is on circle* α *and* C *is on circle* β *and circles* α *and* β *intersect.*

Proof:

1. From the fact A is the center of α it holds that A is inside circle α (using $ax\_generalities3$).
2. From the fact B is the center of β it holds that B is inside circle β (using $ax\_generalities3$).
3. From the facts B is on circle α and A is inside circle α and B is inside circle β and A is on circle β it holds that circles α and β intersect (using ax_rule5).
4. From the fact circles α and β intersect there exist a point C where C is on circle α and C is on circle β (using $ax\_intersections6$).
5. The conclusion follows from the facts C is on circle α and C is on circle β and circles α and β intersect.

QED

Theorem 4 (th_prop1_04.) *Assuming that* $A \neq B$ *and* A *is the center of* α *and* B *is on circle* α *and* B *is the center of* β *and* A *is on circle* β *and* C *is on circle* α *and* C *is on circle* β *and circles* α *and* β *intersect it holds that* $AB \cong AC$*.*

Proof:

1. From the facts A is the center of α and C is on circle α and B is on circle α

2. The conclusion follows from the fact $AB \cong AC$.

QED

Theorem 5 (th_prop1_05.) *Assuming that* $A \neq B$ *and* A *is the center of* α *and* B *is on circle* α *and* B *is the center of* β *and* A *is on circle* β *and* C *is on circle* α *and* C *is on circle* β *and circles* α *and* β *intersect and* $AB \cong AC$ *it holds that* $BA \cong BC$.

Proof:

1. From the facts B is the center of β and C is on circle β and A is on circle β it holds that $BA \cong BC$ (using $ax\_segment3\_2$).
2. The conclusion follows from the fact $BA \cong BC$.

QED

Theorem 6 (th_prop1_06.) *Assuming that* $A \neq B$ *and* A *is the center of* α *and* B *is on circle* α *and* B *is the center of* β *and* A *is on circle* β *and* C *is on circle* α *and* C *is on circle* β *and circles* α *and* β *intersect and* $AB \cong AC$ *and* $BA \cong BC$ *it holds that* $AB \cong BC$ *and* $BC \cong CA$.

Proof:

1. It holds that $AC \cong CA$ (using $ax\_metric3$).
2. It holds that $BA \cong AB$ (using $ax\_metric3$).
3. From the facts $BA \cong AB$ and $BA \cong AB$ it holds that $AB \cong AB$ (using $ax\_cong\_transitivity$).
4. From the facts $AB \cong AC$ and $AB \cong AB$ it holds that $AC \cong AB$ (using $ax\_cong\_transitivity$).
5. From the facts $AC \cong AB$ and $AC \cong CA$ it holds that $AB \cong CA$ (using $ax\_cong\_transitivity$).
6. From the facts $BA \cong AB$ and $BA \cong BC$ it holds that $AB \cong BC$ (using $ax\_cong\_transitivity$).
7. From the facts $AB \cong BC$ and $AB \cong CA$ it holds that $BC \cong CA$ (using $ax\_cong\_transitivity$).
8. The conclusion follows from the facts $AB \cong BC$ and $BC \cong CA$.

QED
