

Examples of proofs in axiomatic system E

Formal system E for Euclid's Elements.
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Theorem 1 (th_prop1aux1_01.) *Assuming that $A \neq B$ and $AB \cong BC$ and $BC \cong CA$ it holds that $C \neq A$ and $C \neq B$.*

Proof:

1. From the fact $AB \cong BC$ it holds that $BC \cong AB$ (using *ax_cong_symmetry*).
2. From the fact $BC \cong CA$ it holds that $CA \cong BC$ (using *ax_cong_symmetry*).
3. From the fact $A \neq B$ it holds that $B \neq A$.
4. It holds that $A = C$ or $A \neq C$.
5. Assume that: $A = C$.
 6. From the facts $CA \cong BC$ and $A = C$ it holds that $AA \cong BA$.
 7. From the fact $AA \cong BA$ it holds that $B = A$ (using *ax_cong_eq1*).
 8. From the facts $B \neq A$ and $B = A$ we get contradiction.
9. Assume that: $A \neq C$.
 10. From the fact $A \neq C$ it holds that $C \neq A$.
 11. It holds that $B = C$ or $B \neq C$.
 12. Assume that: $B = C$.
 13. From the facts $BC \cong AB$ and $B = C$ it holds that $BB \cong AB$.
 14. From the fact $BB \cong AB$ it holds that $A = B$ (using *ax_cong_eq1*).
 15. From the facts $A \neq B$ and $A = B$ we get contradiction.
 16. Assume that: $B \neq C$.
 17. From the fact $B \neq C$ it holds that $C \neq B$.
 18. The conclusion follows from the facts $C \neq A$ and $C \neq B$.
19. The conjecture follows in all cases.
20. The conjecture follows in all cases.

QED
