Examples of proofs in axiomatic system E

Formal system E for Euclid's Elements.
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Theorem 1 (th_prop2_01.) *Assuming that* $B \neq C$ *and* $A \neq B$ *and* $A \neq C$ *there exist point* D*, such that* $D \neq A$ *and* $D \neq B$ *and* $AB \cong BD$ *and* $BD \cong DA$*.*

Theorem 2 (th_prop2_02.) *Assuming that* $A \neq B$ *and* $C \neq A$ *and* $C \neq B$ *and* $D \neq C$ *and* $D \neq A$ *and* $CA \cong AD$ *and* $AD \cong DC$ *there exist line* p*, such that* $on(D, p)$ *and* $on(C, p)$*.*

Proof:

1. From the fact $D \neq C$ it holds that $C \neq D$.
2. From the fact $C \neq D$ there exist a line u where $on(C, u)$ and $on(D, u)$ (using $ax\_lines\_and\_circles1$).
3. The conclusion follows from the facts $on(D, u)$ and $on(C, u)$.

QED

Theorem 3 (th_prop2_03.) *Assuming that* $A \neq B$ *and* $C \neq A$ *and* $C \neq B$ *and* $D \neq C$ *and* $D \neq A$ *and* $CA \cong AD$ *and* $AD \cong DC$ *and* $on(D, p)$ *and* $on(C, p)$ *there exist line* q*, such that* $on(D, q)$ *and* $on(A, q)$*.*

Proof:

1. From the fact $D \neq A$ it holds that $A \neq D$.
2. From the fact $A \neq D$ there exist a line s where $on(A, s)$ and $on(D, s)$ (using $ax\_lines\_and\_circles1$).
3. The conclusion follows from the facts $on(D, s)$ and $on(A, s)$.

QED

Theorem 4 (th_prop2_04.) *Assuming that* $A \neq B$ *and* $C \neq A$ *and* $C \neq B$ *and* $D \neq C$ *and* $D \neq A$ *and* $CA \cong AD$ *and* $AD \cong DC$ *and* $on(D, p)$ *and* $on(C, p)$ *and* $on(D, q)$ *and* $on(A, q)$ *there exist circle* α*, such that* A *is the center of* α *and* B *is on circle* α*.*

Proof:

1. From the fact $A \neq B$ there exist a circle α, where A is the center of α and B is on circle α (using $ax\_lines\_and\_circles2$).
2. The conclusion follows from the facts A is the center of α and B is on circle α.

QED

Theorem 5 (th_prop2_05.) *Assuming that* $A \neq B$ *and* $C \neq A$ *and* $C \neq B$ *and* $D \neq C$ *and* $D \neq A$ *and* $CA \cong AD$ *and* $AD \cong DC$ *and* $on(D, p)$ *and* $on(C, p)$ *and* $on(D, q)$ *and* $on(A, q)$ *and* A *is the center of* α *and* B *is on circle* α *there exist point* E*, such that* $on(E, q)$ *and* E *is on circle* α *and* $bet(D, A, E)$*.*

Proof:

1. From the fact A is the center of α it holds that A is inside circle α (using

2. From the facts A is inside circle α and $on(A, q)$ and $D \neq A$ and $on(D, q)$ there exist a point E where E is on circle α and $on(E, q)$ and $bet(E, A, D)$ (using $ax\_intersections5$).

3. From the fact $bet(E, A, D)$ it holds that $bet(D, A, E)$ and $E \neq A$ and $E \neq D$ and $\neg bet(A, E, D)$ (using ax_bet1).

4. The conclusion follows from the facts $on(E, q)$ and E is on circle α and $bet(D, A, E)$.

QED

Theorem 6 (th_prop2_06.) *Assuming that $A \neq B$ and $C \neq A$ and $C \neq B$ and $D \neq C$ and $D \neq A$ and $CA \cong AD$ and $AD \cong DC$ and $on(D, p)$ and $on(C, p)$ and $on(D, q)$ and $on(A, q)$ and A is the center of α and B is on circle α and $on(E, q)$ and E is on circle α and $bet(D, A, E)$ it holds that $segment\_add(D, A, A, E, D, E)$.*

Proof:

1. From the fact $bet(D, A, E)$ it holds that $segment\_add(D, A, A, E, D, E)$ (using $ax\_segment1$).

2. The conclusion follows from the fact $segment\_add(D, A, A, E, D, E)$.

QED

Theorem 7 (th_prop2_07.) *Assuming that $B \neq C$ and $A \neq B$ and $A \neq C$ and $D \neq A$ and $D \neq B$ and $AB \cong BD$ and $BD \cong DA$ and $on(D, t)$ and $on(A, t)$ and $on(D, u)$ and $on(B, u)$ and B is the center of $\alpha 2$ and C is on circle $\alpha 2$ and $on(I, u)$ and I is on circle $\alpha 2$ and $bet(D, B, I)$ and $segment\_add(D, B, B, I, D, I)$ it holds that $segment\_add(D, I, D, A, B, I)$.*

Theorem 8 (th_prop2_08.) *Assuming that $A \neq B$ and $C \neq A$ and $C \neq B$ and $D \neq C$ and $D \neq A$ and $CA \cong AD$ and $AD \cong DC$ and $on(D, p)$ and $on(C, p)$ and $on(D, q)$ and $on(A, q)$ and A is the center of α and B is on circle α and $on(E, q)$ and E is on circle α and $bet(D, A, E)$ and $segment\_add(D, A, A, E, D, E)$ and $segment\_add(D, E, D, C, A, E)$ it holds that $cong\_less(D, C, D, E)$.*

Proof:

1. It holds that $DA \cong AD$ (using $ax\_metric3$).

2. From the fact $segment\_add(D, A, A, E, D, E)$ it holds that $cong\_less(D, A, D, E)$ and $cong\_less(A, E, D, E)$ (using ax_cong_less1).

3. From the facts $cong\_less(D, A, D, E)$ and $DA \cong AD$ it holds that $cong\_less(A, D, D, E)$ (using ax_cong_less3).

4. From the facts $cong\_less(A, D, D, E)$ and $AD \cong DC$ it holds that $cong\_less(D, C, D, E)$ (using ax_cong_less3).

5. The conclusion follows from the fact $cong\_less(D, C, D, E)$.

QED

Theorem 9 (th_prop2_09.) *Assuming that $A \neq B$ and $C \neq A$ and $C \neq B$ and $D \neq C$ and $D \neq A$ and $CA \cong AD$ and $AD \cong DC$ and $on(D, p)$ and $on(C, p)$ and $on(D, q)$ and $on(A, q)$ and A is the center of α and B is on circle α and $on(E, q)$ and E is on circle α and $bet(D, A, E)$ and $segment\_add(D, A, A, E, D, E)$ and $segment\_add(D, E, D, C, A, E)$ and $cong\_less(D, C, D, E)$ there exist circle β, such that D is the center of β and E is on circle β.*

Proof:

1. From the fact $bet(D, A, E)$ it holds that $bet(E, A, D)$ and $D \neq A$ and $D \neq E$ and $\neg bet(A, D, E)$ (using ax_bet1).

2. From the fact $D \neq E$ there exist a circle $\beta 5$, where D is the center of $\beta 5$ and E is on circle $\beta 5$ (using $ax\_lines\_and\_circles2$).

QED

Theorem 10 (th_prop2_10.) *Assuming that* $A \neq B$ *and* $C \neq A$ *and* $C \neq B$ *and* $D \neq C$ *and* $D \neq A$ *and* $CA \cong AD$ *and* $AD \cong DC$ *and* $on(D,p)$ *and* $on(C,p)$ *and* $on(D,q)$ *and* $on(A,q)$ *and* A *is the center of* α *and* B *is on circle* α *and* $on(E,q)$ *and* E *is on circle* α *and* $bet(D,A,E)$ *and* $segment\_add(D,A,A,E,D,E)$ *and* $segment\_add(D,E,D,C,A,E)$ *and* $cong\_less(D,C,D,E)$ *and* D *is the center of* β *and* E *is on circle* β *it holds that* C *is inside circle* β*.*

Proof:

1. From the facts D is the center of β and E is on circle β and $cong\_less(D,C,D,E)$ it holds that C is inside circle β (using $ax\_segment4\_1$).
2. The conclusion follows from the fact C is inside circle β.

QED

Theorem 11 (th_prop2_11.) *Assuming that* $A \neq B$ *and* $C \neq A$ *and* $C \neq B$ *and* $D \neq C$ *and* $D \neq A$ *and* $CA \cong AD$ *and* $AD \cong DC$ *and* $on(D,p)$ *and* $on(C,p)$ *and* $on(D,q)$ *and* $on(A,q)$ *and* A *is the center of* α *and* B *is on circle* α *and* $on(E,q)$ *and* E *is on circle* α *and* $bet(D,A,E)$ *and* $segment\_add(D,A,A,E,D,E)$ *and* $segment\_add(D,E,D,C,A,E)$ *and* $cong\_less(D,C,D,E)$ *and* D *is the center of* β *and* E *is on circle* β *and* C *is inside circle* β *there exist point* F*, such that* F *is on circle* β *and* $on(F,p)$ *and* $bet(D,C,F)$*.*

Proof:

1. From the facts C is inside circle β and $on(C,p)$ and $D \neq C$ and $on(D,p)$ there exist a point F where F is on circle β and $on(F,p)$ and $bet(F,C,D)$ (using $ax\_intersections5$).
2. From the fact $bet(F,C,D)$ it holds that $bet(D,C,F)$ and $F \neq C$ and $F \neq D$ and $\neg bet(C,F,D)$ (using ax_bet1).
3. The conclusion follows from the facts F is on circle β and $on(F,p)$ and $bet(D,C,F)$.

QED

Theorem 12 (th_prop2_12.) *Assuming that* $A \neq B$ *and* $C \neq A$ *and* $C \neq B$ *and* $D \neq C$ *and* $D \neq A$ *and* $CA \cong AD$ *and* $AD \cong DC$ *and* $on(D,p)$ *and* $on(C,p)$ *and* $on(D,q)$ *and* $on(A,q)$ *and* A *is the center of* α *and* B *is on circle* α *and* $on(E,q)$ *and* E *is on circle* α *and* $bet(D,A,E)$ *and* $segment\_add(D,A,A,E,D,E)$ *and* $segment\_add(D,E,D,C,A,E)$ *and* $cong\_less(D,C,D,E)$ *and* D *is the center of* β *and* E *is on circle* β *and* C *is inside circle* β *and* F *is on circle* β *and* $on(F,p)$ *and* $bet(D,C,F)$ *it holds that* $segment\_add(D,C,C,F,D,F)$*.*

Proof:

1. From the fact $bet(D,C,F)$ it holds that $segment\_add(D,C,C,F,D,F)$ (using $ax\_segment1$).
2. The conclusion follows from the fact $segment\_add(D,C,C,F,D,F)$.

QED

Theorem 13 (th_prop2_13.) *Assuming that* $A \neq B$ *and* $C \neq A$ *and* $C \neq B$ *and* $D \neq C$ *and* $D \neq A$ *and* $CA \cong AD$ *and* $AD \cong DC$ *and* $on(D,p)$ *and* $on(C,p)$ *and* $on(D,q)$ *and* $on(A,q)$ *and* A *is the center of* α *and* B *is on circle* α *and* $on(E,q)$ *and* E *is on circle* α *and* $bet(D,A,E)$ *and* $segment\_add(D,A,A,E,D,E)$ *and* $segment\_add(D,E,D,C,A,E)$ *and* $cong\_less(D,C,D,E)$ *and* D *is the center of* β *and* E *is on circle* β *and* C *is inside circle* β *and* F *is on circle* β *and* $on(F,p)$ *and* $bet(D,C,F)$ *and* $segment\_add(D,C,C,F,D,F)$ *it holds that* $DF \cong DE$*.*

Proof:

1. From the facts D is the center of β and E is on circle β and F is on circle β

2. The conclusion follows from the fact $DF \cong DE$.

QED

Theorem 14 (th_prop2_14.) *Assuming that $B \neq C$ and $A \neq B$ and $A \neq C$ and $D \neq A$ and $D \neq B$ and $AB \cong BD$ and $BD \cong DA$ and $on(D,t)$ and $on(A,t)$ and $on(D,u)$ and $on(B,u)$ and B is the center of $\alpha 2$ and C is on circle $\alpha 2$ and $on(I,u)$ and I is on circle $\alpha 2$ and $bet(D,B,I)$ and $segment\_add(D,B,B,I,D,I)$ and $segment\_add(D,I,D,A,B,I)$ and $cong\_less(D,A,D,I)$ and D is the center of $\alpha 3$ and I is on circle $\alpha 3$ and A is inside circle $\alpha 3$ and K is on circle $\alpha 3$ and $on(K,t)$ and $bet(D,A,K)$ and $segment\_add(D,A,A,K,D,K)$ and $DK \cong DI$ it holds that $segment\_add(D,A,B,I,D,K)$.*

Theorem 15 (th_prop2_15.) *Assuming that $B \neq C$ and $A \neq B$ and $A \neq C$ and $D \neq A$ and $D \neq B$ and $AB \cong BD$ and $BD \cong DA$ and $on(D,t)$ and $on(A,t)$ and $on(D,u)$ and $on(B,u)$ and B is the center of $\alpha 2$ and C is on circle $\alpha 2$ and $on(I,u)$ and I is on circle $\alpha 2$ and $bet(D,B,I)$ and $segment\_add(D,B,B,I,D,I)$ and $segment\_add(D,I,D,A,B,I)$ and $cong\_less(D,A,D,I)$ and D is the center of $\alpha 3$ and I is on circle $\alpha 3$ and A is inside circle $\alpha 3$ and K is on circle $\alpha 3$ and $on(K,t)$ and $bet(D,A,K)$ and $segment\_add(D,A,A,K,D,K)$ and $DK \cong DI$ and $segment\_add(D,A,B,I,D,K)$ it holds that $AK \cong BI$.*

Theorem 16 (th_prop2_16.) *Assuming that $A \neq B$ and $C \neq A$ and $C \neq B$ and $D \neq C$ and $D \neq A$ and $CA \cong AD$ and $AD \cong DC$ and $on(D,p)$ and $on(C,p)$ and $on(D,q)$ and $on(A,q)$ and A is the center of α and B is on circle α and $on(E,q)$ and E is on circle α and $bet(D,A,E)$ and $segment\_add(D,A,A,E,D,E)$ and $segment\_add(D,E,D,C,A,E)$ and $cong\_less(D,C,D,E)$ and D is the center of β and E is on circle β and C is inside circle β and F is on circle β and $on(F,p)$ and $bet(D,C,F)$ and $segment\_add(D,C,C,F,D,F)$ and $DF \cong DE$ and $segment\_add(D,C,A,E,D,F)$ and $CF \cong AE$ it holds that $AE \cong AB$.*

Proof:

1. From the facts A is the center of α and B is on circle α and E is on circle α it holds that $AE \cong AB$ (using $ax\_segment3\_2$).
2. The conclusion follows from the fact $AE \cong AB$.

QED

Theorem 17 (th_prop2_17.) *Assuming that $A \neq B$ and $C \neq A$ and $C \neq B$ and $D \neq C$ and $D \neq A$ and $CA \cong AD$ and $AD \cong DC$ and $on(D,p)$ and $on(C,p)$ and $on(D,q)$ and $on(A,q)$ and A is the center of α and B is on circle α and $on(E,q)$ and E is on circle α and $bet(D,A,E)$ and $segment\_add(D,A,A,E,D,E)$ and $segment\_add(D,E,D,C,A,E)$ and $cong\_less(D,C,D,E)$ and D is the center of β and E is on circle β and C is inside circle β and F is on circle β and $on(F,p)$ and $bet(D,C,F)$ and $segment\_add(D,C,C,F,D,F)$ and $DF \cong DE$ and $segment\_add(D,C,A,E,D,F)$ and $CF \cong AE$ and $AE \cong AB$ it holds that $CF \cong AB$.*

Proof:

1. From the fact $CF \cong AE$ it holds that $AE \cong CF$ (using $ax\_cong\_symmetry$).
2. From the facts $AE \cong AB$ and $AE \cong CF$ it holds that $AB \cong CF$ (using $ax\_cong\_transitivity$).
3. From the fact $AB \cong CF$ it holds that $CF \cong AB$ (using $ax\_cong\_symmetry$).
4. The conclusion follows from the fact $CF \cong AB$.

QED
