High school geometry theorems

Hilbert's axiomatic system.
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07.01.2017.

Theorem 1 (th_11_01.) *Assuming that $A \notin p$ and $A \in q$ and lines p and q intersect there exist plane α, such that $A \in \alpha$ and $p \in \alpha$.*

Proof:

1. There exist a point B and a point C where $B \neq C$ and $B \in p$ and $C \in p$ (using ax_I3a).
2. From the facts $B \neq C$ and $B \in p$ and $C \in p$ and $A \notin p$ it holds that $\neg col(B, C, A)$ (using ax_D1a).
3. From the fact $\neg col(B, C, A)$ it holds that $\neg col(C, A, B)$ (using ax_sym_ncol1).
4. From the fact $\neg col(C, A, B)$ it holds that $\neg col(A, B, C)$ (using ax_sym_ncol1).
5. From the fact $\neg col(A, B, C)$ there exist a plane α, where $A \in \alpha$ and $B \in \alpha$ and $C \in \alpha$ (using ax_I4a).
6. From the facts $B \neq C$ and $B \in p$ and $C \in p$ and $B \in \alpha$ and $C \in \alpha$ it holds that $p \in \alpha$ (using ax_I6).
7. The conclusion follows from the facts $A \in \alpha$ and $p \in \alpha$.

QED

Theorem 2 (th_11_02.) *Assuming that $A \notin p$ and $A \in q$ and lines p and q intersect and $A \in \alpha$ and $p \in \alpha$ it holds that $q \in \alpha$.*

Proof:

1. From the fact lines p and q intersect there exist a point B where $p \neq q$ and $B \in p$ and $B \in q$ (using ax_D6).
2. From the facts $p \in \alpha$ and $B \in p$ it holds that $B \in \alpha$ (using ax_D11).
3. It holds that $A = B$ or $A \neq B$.
   4. Assume that: $A = B$.
      5. From the facts $B \in p$ and $A = B$ it holds that $A \in p$.
      6. From the facts $A \notin p$ and $A \in p$ we get contradiction.
   7. Assume that: $A \neq B$.
      8. From the facts $A \neq B$ and $A \in q$ and $B \in q$ and $A \in \alpha$ and $B \in \alpha$ it holds that $q \in \alpha$ (using ax_I6).
      9. The conclusion follows from the fact $q \in \alpha$.
10. The conjecture follows in all cases.

QED
