High school geometry theorems

Hilbert's axiomatic system.
Formalized by: Sana Stojanovic Djurdjevic
Proved by: ArgoGeoChecker.

07.01.2017.

Theorem 1 (th_12_01.) *Assuming that $A \notin p$ and $A \notin q$ and $r \neq s$ and $A \in r$ and $A \in s$ and lines r and p intersect and lines r and q intersect and lines s and p intersect and lines s and q intersect there exist point B, such that $B \in r$ and $B \in p$.*

Proof:

1. From the fact lines r and p intersect there exist a point B where $r \neq p$ and $B \in r$ and $B \in p$ (using ax_D6).
2. The conclusion follows from the facts $B \in r$ and $B \in p$.

QED

Theorem 2 (th_12_02.) *Assuming that $A \notin p$ and $A \notin q$ and $r \neq s$ and $A \in r$ and $A \in s$ and lines r and p intersect and lines r and q intersect and lines s and p intersect and lines s and q intersect and $B \in r$ and $B \in p$ there exist point C, such that $C \in s$ and $C \in p$.*

Proof:

1. From the fact lines s and p intersect there exist a point C where $s \neq p$ and $C \in s$ and $C \in p$ (using ax_D6).
2. The conclusion follows from the facts $C \in s$ and $C \in p$.

QED

Theorem 3 (th_12_03.) *Assuming that $A \notin p$ and $A \notin q$ and $r \neq s$ and $A \in r$ and $A \in s$ and lines r and p intersect and lines r and q intersect and lines s and p intersect and lines s and q intersect and $B \in r$ and $B \in p$ and $C \in s$ and $C \in p$ there exist point D, such that $D \in r$ and $D \in q$.*

Proof:

1. From the fact lines r and q intersect there exist a point D where $r \neq q$ and $D \in r$ and $D \in q$ (using ax_D6).
2. The conclusion follows from the facts $D \in r$ and $D \in q$.

QED

Theorem 4 (th_12_04.) *Assuming that $A \notin p$ and $A \notin q$ and $r \neq s$ and $A \in r$ and $A \in s$ and lines r and p intersect and lines r and q intersect and lines s and p intersect and lines s and q intersect and $B \in r$ and $B \in p$ and $C \in s$ and $C \in p$ and $D \in r$ and $D \in q$ there exist point E, such that $E \in s$ and $E \in q$.*

Proof:

1. From the fact lines s and q intersect there exist a point E where $s \neq q$ and $E \in s$ and $E \in q$ (using ax_D6).
2. The conclusion follows from the facts $E \in s$ and $E \in q$.

Theorem 5 (th_12_05.) *Assuming that $A \notin p$ and $A \notin q$ and $r \neq s$ and $A \in r$ and $A \in s$ and lines r and p intersect and lines r and q intersect and lines s and p intersect and lines s and q intersect and $B \in r$ and $B \in p$ and $C \in s$ and $C \in p$ and $D \in r$ and $D \in q$ and $E \in s$ and $E \in q$ it holds that $\neg col(A, B, C)$.*

Proof:

1. It holds that $A = B$ or $A \neq B$.
   2. Assume that: $A = B$.
      3. From the facts $B \in p$ and $A = B$ it holds that $A \in p$.
      4. From the facts $A \notin p$ and $A \in p$ we get contradiction.
   5. Assume that: $A \neq B$.
      6. It holds that $B = C$ or $B \neq C$.
         7. Assume that: $B = C$.
            8. From the facts $C \in s$ and $B = C$ it holds that $B \in s$.
            9. From the facts $A \neq B$ and $A \in r$ and $B \in r$ and $A \in s$ and $B \in s$ it holds that $r = s$ (using ax_I2).
            10. From the facts $r \neq s$ and $r = s$ we get contradiction.
         11. Assume that: $B \neq C$.
            12. From the facts $B \neq C$ and $B \in p$ and $C \in p$ and $A \notin p$ it holds that $\neg col(B, C, A)$ (using ax_D1a).
            13. From the fact $\neg col(B, C, A)$ it holds that $\neg col(B, A, C)$ and $\neg col(C, B, A)$ and $\neg col(C, A, B)$ and $\neg col(A, B, C)$ and $\neg col(A, C, B)$ (using ax_sym_ncol).
            14. The conclusion follows from the fact $\neg col(A, B, C)$.
      15. The conjecture follows in all cases.
16. The conjecture follows in all cases.

QED

Theorem 6 (th_12_06.) *Assuming that $A \notin p$ and $A \notin q$ and $r \neq s$ and $A \in r$ and $A \in s$ and lines r and p intersect and lines r and q intersect and lines s and p intersect and lines s and q intersect and $B \in r$ and $B \in p$ and $C \in s$ and $C \in p$ and $D \in r$ and $D \in q$ and $E \in s$ and $E \in q$ and $\neg col(A, B, C)$ there exist plane α, such that $A \in \alpha$ and $B \in \alpha$ and $C \in \alpha$.*

Proof:

1. From the fact $\neg col(A, B, C)$ there exist a plane α, where $A \in \alpha$ and $B \in \alpha$ and $C \in \alpha$ (using ax_I4a).
2. The conclusion follows from the facts $A \in \alpha$ and $B \in \alpha$ and $C \in \alpha$.

QED

Theorem 7 (th_12_07.) *Assuming that $A \notin p$ and $A \notin q$ and $r \neq s$ and $A \in r$ and $A \in s$ and lines r and p intersect and lines r and q intersect and lines s and p intersect and lines s and q intersect and $B \in r$ and $B \in p$ and $C \in s$ and $C \in p$ and $D \in r$ and $D \in q$ and $E \in s$ and $E \in q$ and $\neg col(A, B, C)$ and $A \in \alpha$ and $B \in \alpha$ and $C \in \alpha$ it holds that $r \in \alpha$ and $s \in \alpha$.*

Proof:

1. It holds that $A = B$ or $A \neq B$.
   2. Assume that: $A = B$.
      3. From the facts $B \in p$ and $A = B$ it holds that $A \in p$.
      4. From the facts $A \notin p$ and $A \in p$ we get contradiction.
   5. Assume that: $A \neq B$.
      6. From the facts $A \neq B$ and $A \in r$ and $B \in r$ and $A \in \alpha$ and $B \in \alpha$ it

7. It holds that $A = C$ or $A \neq C$.
8. Assume that: $A = C$.
9. From the facts $C \in p$ and $A = C$ it holds that $A \in p$.
10. From the facts $A \notin p$ and $A \in p$ we get contradiction.
11. Assume that: $A \neq C$.
12. From the facts $A \neq C$ and $A \in s$ and $C \in s$ and $A \in \alpha$ and $C \in \alpha$ it holds that $s \in \alpha$ (using ax_I6).
13. The conclusion follows from the facts $r \in \alpha$ and $s \in \alpha$.
14. The conjecture follows in all cases.
15. The conjecture follows in all cases.

QED

Theorem 8 (th_12_08.) *Assuming that $A \notin p$ and $A \notin q$ and $r \neq s$ and $A \in r$ and $A \in s$ and lines r and p intersect and lines r and q intersect and lines s and p intersect and lines s and q intersect and $B \in r$ and $B \in p$ and $C \in s$ and $C \in p$ and $D \in r$ and $D \in q$ and $E \in s$ and $E \in q$ and $\neg col(A, B, C)$ and $A \in \alpha$ and $B \in \alpha$ and $C \in \alpha$ and $r \in \alpha$ and $s \in \alpha$ it holds that $p \in \alpha$ and $q \in \alpha$.*

Proof:

1. From the facts $r \in \alpha$ and $D \in r$ it holds that $D \in \alpha$ (using ax_D11).
2. From the facts $s \in \alpha$ and $E \in s$ it holds that $E \in \alpha$ (using ax_D11).
3. From the facts $A \in r$ and $B \in r$ and $B \in r$ it holds that $col(A, B, B)$ (using ax_D1).
4. It holds that $A = D$ or $A \neq D$.
5. Assume that: $A = D$.
6. From the facts $D \in q$ and $A = D$ it holds that $A \in q$.
7. From the facts $A \notin q$ and $A \in q$ we get contradiction.
8. Assume that: $A \neq D$.
9. It holds that $B = C$ or $B \neq C$.
10. Assume that: $B = C$.
11. From the facts $col(A, B, B)$ and $B = C$ it holds that $col(A, B, C)$.
12. From the facts $\neg col(A, B, C)$ and $col(A, B, C)$ we get contradiction.
13. Assume that: $B \neq C$.
14. From the facts $B \neq C$ and $B \in p$ and $C \in p$ and $B \in \alpha$ and $C \in \alpha$ it holds that $p \in \alpha$ (using ax_I6).
15. It holds that $D = E$ or $D \neq E$.
16. Assume that: $D = E$.
17. From the facts $E \in s$ and $D = E$ it holds that $D \in s$.
18. From the facts $A \neq D$ and $A \in r$ and $D \in r$ and $A \in s$ and $D \in s$ it holds that $r = s$ (using ax_I2).
19. From the facts $r \neq s$ and $r = s$ we get contradiction.
20. Assume that: $D \neq E$.
21. From the facts $D \neq E$ and $D \in q$ and $E \in q$ and $D \in \alpha$ and $E \in \alpha$ it holds that $q \in \alpha$ (using ax_I6).
22. The conclusion follows from the facts $p \in \alpha$ and $q \in \alpha$.
23. The conjecture follows in all cases.
24. The conjecture follows in all cases.
25. The conjecture follows in all cases.

QED