High school geometry theorems

Hilbert's axiomatic system.
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Theorem 1 (th_14_01.) *Assuming that $A \notin \alpha$ and $B \notin \alpha$ and $C \notin \alpha$ and $\neg col(A, B, C)$ and $A \in p$ and $B \in p$ and $B \in q$ and $C \in q$ and $C \in r$ and $A \in r$ and $D \in \alpha$ and $D \in p$ and $E \in \alpha$ and $E \in q$ and $F \in \alpha$ and $F \in r$ there exist plane β, such that $A \in \beta$ and $B \in \beta$ and $C \in \beta$.*

Proof:

1. From the fact $\neg col(A, B, C)$ there exist a plane β, where $A \in \beta$ and $B \in \beta$ and $C \in \beta$ (using ax_I4a).
2. The conclusion follows from the facts $A \in \beta$ and $B \in \beta$ and $C \in \beta$.

QED

Theorem 2 (th_14_02.) *Assuming that $A \notin \alpha$ and $B \notin \alpha$ and $C \notin \alpha$ and $\neg col(A, B, C)$ and $A \in p$ and $B \in p$ and $B \in q$ and $C \in q$ and $C \in r$ and $A \in r$ and $D \in \alpha$ and $D \in p$ and $E \in \alpha$ and $E \in q$ and $F \in \alpha$ and $F \in r$ and $A \in \beta$ and $B \in \beta$ and $C \in \beta$ it holds that $p \in \beta$.*

Proof:

1. From the facts $A \in r$ and $A \in r$ and $C \in r$ it holds that $col(A, A, C)$ (using ax_D1).
2. It holds that $A = B$ or $A \neq B$.
   3. Assume that: $A = B$.
      4. From the facts $col(A, A, C)$ and $A = B$ it holds that $col(A, B, C)$.
      5. From the facts $\neg col(A, B, C)$ and $col(A, B, C)$ we get contradiction.
   6. Assume that: $A \neq B$.
      7. From the facts $A \neq B$ and $A \in p$ and $B \in p$ and $A \in \beta$ and $B \in \beta$ it holds that $p \in \beta$ (using ax_I6).
      8. The conclusion follows from the fact $p \in \beta$.
9. The conjecture follows in all cases.

QED

Theorem 3 (th_14_03.) *Assuming that $A \notin \alpha$ and $B \notin \alpha$ and $C \notin \alpha$ and $\neg col(A, B, C)$ and $A \in p$ and $B \in p$ and $B \in q$ and $C \in q$ and $C \in r$ and $A \in r$ and $D \in \alpha$ and $D \in p$ and $E \in \alpha$ and $E \in q$ and $F \in \alpha$ and $F \in r$ and $A \in \beta$ and $B \in \beta$ and $C \in \beta$ and $p \in \beta$ it holds that $q \in \beta$.*

Proof:

1. From the facts $A \in p$ and $B \in p$ and $B \in p$ it holds that $col(A, B, B)$ (using ax_D1).
2. It holds that $B = C$ or $B \neq C$.
   3. Assume that: $B = C$.
      4. From the facts $col(A, B, B)$ and $B = C$ it holds that $col(A, B, C)$.

6. Assume that: $B \neq C$.

7. From the facts $B \neq C$ and $B \in q$ and $C \in q$ and $B \in \beta$ and $C \in \beta$ it holds that $q \in \beta$ (using ax_I6).

8. The conclusion follows from the fact $q \in \beta$.

9. The conjecture follows in all cases.

QED

Theorem 4 (th_14_04.) *Assuming that* $A \notin \alpha$ *and* $B \notin \alpha$ *and* $C \notin \alpha$ *and* $\neg col(A, B, C)$ *and* $A \in p$ *and* $B \in p$ *and* $B \in q$ *and* $C \in q$ *and* $C \in r$ *and* $A \in r$ *and* $D \in \alpha$ *and* $D \in p$ *and* $E \in \alpha$ *and* $E \in q$ *and* $F \in \alpha$ *and* $F \in r$ *and* $A \in \beta$ *and* $B \in \beta$ *and* $C \in \beta$ *and* $p \in \beta$ *and* $q \in \beta$ *it holds that* $r \in \beta$.

Proof:

1. From the facts $A \in p$ and $B \in p$ and $A \in p$ it holds that $col(A, B, A)$ (using ax_D1).

2. It holds that $A = C$ or $A \neq C$.

3. Assume that: $A = C$.

4. From the facts $col(A, B, A)$ and $A = C$ it holds that $col(A, B, C)$.

5. From the facts $\neg col(A, B, C)$ and $col(A, B, C)$ we get contradiction.

6. Assume that: $A \neq C$.

7. From the facts $A \neq C$ and $A \in r$ and $C \in r$ and $A \in \beta$ and $C \in \beta$ it holds that $r \in \beta$ (using ax_I6).

8. The conclusion follows from the fact $r \in \beta$.

9. The conjecture follows in all cases.

QED

Theorem 5 (th_14_05.) *Assuming that* $A \notin \alpha$ *and* $B \notin \alpha$ *and* $C \notin \alpha$ *and* $\neg col(A, B, C)$ *and* $A \in p$ *and* $B \in p$ *and* $B \in q$ *and* $C \in q$ *and* $C \in r$ *and* $A \in r$ *and* $D \in \alpha$ *and* $D \in p$ *and* $E \in \alpha$ *and* $E \in q$ *and* $F \in \alpha$ *and* $F \in r$ *and* $A \in \beta$ *and* $B \in \beta$ *and* $C \in \beta$ *and* $p \in \beta$ *and* $q \in \beta$ *and* $r \in \beta$ *it holds that* $D \in \beta$ *and* $E \in \beta$ *and* $F \in \beta$.

Proof:

1. From the facts $p \in \beta$ and $D \in p$ it holds that $D \in \beta$ (using ax_D11).
2. From the facts $q \in \beta$ and $E \in q$ it holds that $E \in \beta$ (using ax_D11).
3. From the facts $r \in \beta$ and $F \in r$ it holds that $F \in \beta$ (using ax_D11).
4. The conclusion follows from the facts $D \in \beta$ and $E \in \beta$ and $F \in \beta$.

QED

Theorem 6 (th_14_06.) *Assuming that* $B \notin \alpha$ *and* $C \notin \alpha$ *and* $D \notin \alpha$ *and* $\neg col(B, C, D)$ *and* $B \in t$ *and* $C \in t$ *and* $C \in u$ *and* $D \in u$ *and* $D \in v$ *and* $B \in v$ *and* $I \in \alpha$ *and* $I \in t$ *and* $J \in \alpha$ *and* $J \in u$ *and* $K \in \alpha$ *and* $K \in v$ *and* $B \in \gamma 3$ *and* $C \in \gamma 3$ *and* $D \in \gamma 3$ *and* $t \in \gamma 3$ *and* $u \in \gamma 3$ *and* $v \in \gamma 3$ *and* $I \in \gamma 3$ *and* $J \in \gamma 3$ *and* $K \in \gamma 3$ *there exist line* $t1$*, such that* $t1 \in \alpha$ *and* $t1 \in \gamma 3$.

Theorem 7 (th_14_07.) *Assuming that* $B \notin \alpha$ *and* $C \notin \alpha$ *and* $D \notin \alpha$ *and* $\neg col(B, C, D)$ *and* $B \in t$ *and* $C \in t$ *and* $C \in u$ *and* $D \in u$ *and* $D \in v$ *and* $B \in v$ *and* $I \in \alpha$ *and* $I \in t$ *and* $J \in \alpha$ *and* $J \in u$ *and* $K \in \alpha$ *and* $K \in v$ *and* $B \in \gamma 3$ *and* $C \in \gamma 3$ *and* $D \in \gamma 3$ *and* $t \in \gamma 3$ *and* $u \in \gamma 3$ *and* $v \in \gamma 3$ *and* $I \in \gamma 3$ *and* $J \in \gamma 3$ *and* $K \in \gamma 3$ *and* $t1 \in \alpha$ *and* $t1 \in \gamma 3$ *there exist point* N*, point* P*, such that* $N \neq P$ *and* $N \in t1$ *and* $P \in t1$.

Theorem 8 (th_14_08.) *Assuming that* $B \notin \alpha$ *and* $C \notin \alpha$ *and* $D \notin \alpha$ *and* $\neg col(B, C, D)$ *and* $B \in t$ *and* $C \in t$ *and* $C \in u$ *and* $D \in u$ *and* $D \in v$ *and* $B \in v$ *and* $I \in \alpha$ *and* $I \in t$ *and* $J \in \alpha$ *and* $J \in u$ *and* $K \in \alpha$ *and* $K \in v$ *and* $B \in \gamma 3$ *and* $C \in \gamma 3$ *and* $D \in \gamma 3$ *and* $t \in \gamma 3$ *and* $u \in \gamma 3$ *and* $v \in \gamma 3$ *and* $I \in \gamma 3$ *and* $J \in \gamma 3$ *and* $K \in \gamma 3$ *and* $t1 \in \alpha$ *and* $t1 \in \gamma 3$ *and* $N \neq P$ *and* $N \in t1$ *and* $P \in t1$ *it holds that* $I \in t1$ *and* $J \in t1$ *and* $K \in t1$.

Theorem 9 (th_14_09.) *Assuming that* $A \notin \alpha$ *and* $B \notin \alpha$ *and* $C \notin \alpha$ *and* $\neg col(A, B, C)$ *and* $A \in p$ *and* $B \in p$ *and* $B \in q$ *and* $C \in q$ *and* $C \in r$ *and* $A \in r$ *and* $D \in \alpha$ *and* $D \in p$ *and* $E \in \alpha$ *and* $E \in q$ *and* $F \in \alpha$ *and* $F \in r$ *and* $A \in \beta$ *and* $B \in \beta$ *and* $C \in \beta$ *and* $p \in \beta$ *and* $q \in \beta$ *and* $r \in \beta$ *and* $D \in \beta$ *and* $E \in \beta$ *and* $F \in \beta$ *and* $s \in \alpha$ *and* $s \in \beta$ *and* $G \neq I$ *and* $G \in s$ *and* $I \in s$ *and* $D \in s$ *and* $E \in s$ *and* $F \in s$ *it holds that* $col(D, E, F)$.

Proof:

1. From the facts $D \in s$ and $E \in s$ and $F \in s$ it holds that $col(D, E, F)$ (using ax_D1).
2. The conclusion follows from the fact $col(D, E, F)$.

QED
