High school geometry theorems

Hilbert's axiomatic system.
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Theorem 1 (th_15_01.) *Assuming that* $bet(A,B,C)$ *and* $bet(A,D,C)$ *there exist line* p*, such that* $A \in p$ *and* $B \in p$ *and* $C \in p$ *and* $D \in p$.

Proof:

1. From the fact $bet(A,B,C)$ it holds that $A \neq B$ and $A \neq C$ and $B \neq C$ and $col(A,B,C)$ and $bet(C,B,A)$ (using ax_II1).
2. From the fact $bet(A,D,C)$ it holds that $A \neq D$ and $A \neq C$ and $D \neq C$ and $col(A,D,C)$ and $bet(C,D,A)$ (using ax_II1).
3. From the fact $col(A,D,C)$ it holds that $col(A,C,D)$ and $col(D,A,C)$ and $col(D,C,A)$ and $col(C,A,D)$ and $col(C,D,A)$ (using ax_sym_col).
4. From the fact $col(A,B,C)$ there exist a line p where $A \in p$ and $B \in p$ and $C \in p$ (using ax_D2).
5. From the fact $col(A,C,D)$ there exist a line q where $A \in q$ and $C \in q$ and $D \in q$ (using ax_D2).
6. From the facts $A \neq C$ and $A \in p$ and $C \in p$ and $A \in q$ and $C \in q$ it holds that $p = q$ (using ax_I2).
7. From the facts $D \in q$ and $p = q$ it holds that $D \in p$.
8. The conclusion follows from the facts $A \in p$ and $B \in p$ and $C \in p$ and $D \in p$.

QED

Theorem 2 (th_15_02.) *Assuming that* $bet(A,B,C)$ *and* $bet(A,D,C)$ *and* $A \in p$ *and* $B \in p$ *and* $C \in p$ *and* $D \in p$ *it holds that* $col(B,A,D)$.

Proof:

1. From the facts $B \in p$ and $A \in p$ and $D \in p$ it holds that $col(B,A,D)$ (using ax_D1).
2. The conclusion follows from the fact $col(B,A,D)$.

QED

Theorem 3 (th_15_03.) *Assuming that* $bet(A,B,C)$ *and* $bet(A,D,C)$ *and* $A \in t$ *and* $B \in t$ *and* $C \in t$ *and* $D \in t$ *and* $col(B,A,D)$ *it holds that* $\neg bet(B,A,D)$.
