

# High school geometry theorems

Hilbert's axiomatic system.  
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**Theorem 1 (th\_17\_01.)** *Assuming that  $bet(A, B, C)$  and  $bet(C, B, D)$  and  $bet(D, B, E)$  there exist line  $p$ , such that  $A \in p$  and  $C \in p$  and  $D \in p$  and  $E \in p$  and  $B \in p$ .*

*Proof:*

1. From the fact  $bet(A, B, C)$  it holds that  $A \neq B$  and  $A \neq C$  and  $B \neq C$  and  $col(A, B, C)$  and  $bet(C, B, A)$  (using  $ax\_II1$ ).
2. From the fact  $bet(C, B, D)$  it holds that  $C \neq B$  and  $C \neq D$  and  $B \neq D$  and  $col(C, B, D)$  and  $bet(D, B, C)$  (using  $ax\_II1$ ).
3. From the fact  $bet(D, B, C)$  it holds that  $D \neq B$  and  $D \neq C$  and  $B \neq C$  and  $col(D, B, C)$  and  $bet(C, B, D)$  (using  $ax\_II1$ ).
4. From the fact  $bet(D, B, E)$  it holds that  $D \neq B$  and  $D \neq E$  and  $B \neq E$  and  $col(D, B, E)$  and  $bet(E, B, D)$  (using  $ax\_II1$ ).
5. From the fact  $bet(E, B, D)$  it holds that  $E \neq B$  and  $E \neq D$  and  $B \neq D$  and  $col(E, B, D)$  and  $bet(D, B, E)$  (using  $ax\_II1$ ).
6. From the fact  $col(C, B, D)$  it holds that  $col(C, D, B)$  and  $col(B, C, D)$  and  $col(B, D, C)$  and  $col(D, C, B)$  and  $col(D, B, C)$  (using  $ax\_sym\_col$ ).
7. From the fact  $col(D, B, E)$  it holds that  $col(D, E, B)$  and  $col(B, D, E)$  and  $col(B, E, D)$  and  $col(E, D, B)$  and  $col(E, B, D)$  (using  $ax\_sym\_col$ ).
8. From the fact  $col(A, B, C)$  there exist a line  $p$  where  $A \in p$  and  $B \in p$  and  $C \in p$  (using  $ax\_D2$ ).
9. From the fact  $col(B, C, D)$  there exist a line  $q$  where  $B \in q$  and  $C \in q$  and  $D \in q$  (using  $ax\_D2$ ).
10. From the fact  $col(B, D, E)$  there exist a line  $r$  where  $B \in r$  and  $D \in r$  and  $E \in r$  (using  $ax\_D2$ ).
11. It holds that  $A = D$  or  $A \neq D$ .
12. Assume that:  $A = D$ .
13. From the facts  $col(D, B, E)$  and  $A = D$  it holds that  $col(A, B, E)$ .
14. From the fact  $col(A, B, E)$  there exist a line  $s$  where  $A \in s$  and  $B \in s$  and  $E \in s$  (using  $ax\_D2$ ).
15. From the facts  $A \neq B$  and  $A \in p$  and  $B \in p$  and  $A \in s$  and  $B \in s$  it holds that  $p = s$  (using  $ax\_I2$ ).
16. From the facts  $A \in p$  and  $A = D$  it holds that  $D \in p$ .
17. From the facts  $E \in s$  and  $p = s$  it holds that  $E \in p$ .
18. The conclusion follows from the facts  $A \in p$  and  $C \in p$  and  $D \in p$  and  $E \in p$  and  $B \in p$ .
19. Assume that:  $A \neq D$ .
20. From the facts  $B \neq C$  and  $B \in p$  and  $C \in p$  and  $B \in q$  and  $C \in q$  it holds that  $p = q$  (using  $ax\_I2$ ).
21. From the facts  $D \in q$  and  $p = q$  it holds that  $D \in p$ .
22. From the facts  $B \neq D$  and  $B \in p$  and  $D \in p$  and  $B \in r$  and  $D \in r$  it holds that  $p = r$  (using  $ax\_I2$ ).

- 23. From the facts  $D \in q$  and  $p = q$  and  $p = r$  it holds that  $D \in p$ .
- 24. From the facts  $E \in r$  and  $p = q$  and  $p = r$  it holds that  $E \in p$ .
- 25. The conclusion follows from the facts  $A \in p$  and  $C \in p$  and  $D \in p$  and  $E \in p$  and  $B \in p$ .
- 26. The conjecture follows in all cases.

QED

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**Theorem 2 (th\_17\_02.)** *Assuming that  $\text{bet}(A, E, B)$  and  $\text{bet}(B, E, C)$  and  $\text{bet}(C, E, D)$  and  $A \in u$  and  $B \in u$  and  $C \in u$  and  $D \in u$  and  $E \in u$  it holds that  $\text{bet}(A, E, D)$ .*

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