High school geometry theorems

Hilbert's axiomatic system.
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Theorem 1 (th_18_01.) *Assuming that* $bet(A, B, C)$ *and* $bet(A, D, C)$ *there exist line* p*, such that* $A \in p$ *and* $B \in p$ *and* $C \in p$ *and* $D \in p$*.*

Proof:

1. From the fact $bet(A, B, C)$ it holds that $A \neq B$ and $A \neq C$ and $B \neq C$ and $col(A, B, C)$ and $bet(C, B, A)$ (using ax_II1).
2. From the fact $bet(A, D, C)$ it holds that $A \neq D$ and $A \neq C$ and $D \neq C$ and $col(A, D, C)$ and $bet(C, D, A)$ (using ax_II1).
3. From the fact $col(A, D, C)$ it holds that $col(A, C, D)$ and $col(D, A, C)$ and $col(D, C, A)$ and $col(C, A, D)$ and $col(C, D, A)$ (using ax_sym_col).
4. From the fact $col(A, B, C)$ there exist a line p where $A \in p$ and $B \in p$ and $C \in p$ (using ax_D2).
5. From the fact $col(A, C, D)$ there exist a line q where $A \in q$ and $C \in q$ and $D \in q$ (using ax_D2).
6. From the facts $A \neq C$ and $A \in p$ and $C \in p$ and $A \in q$ and $C \in q$ it holds that $p = q$ (using ax_I2).
7. From the facts $D \in q$ and $p = q$ it holds that $D \in p$.
8. The conclusion follows from the facts $A \in p$ and $B \in p$ and $C \in p$ and $D \in p$.

QED

Theorem 2 (th_18_02.) *Assuming that* $bet(A, B, C)$ *and* $bet(A, D, C)$ *and* $A \in t$ *and* $B \in t$ *and* $C \in t$ *and* $D \in t$ *it holds that* $bet(A, B, D)$*.*

Theorem 3 (th_18_03.) *Assuming that* $bet(A, B, C)$ *and* $bet(A, D, C)$ *and* $A \in t$ *and* $B \in t$ *and* $C \in t$ *and* $D \in t$ *and* $bet(A, B, D)$ *it holds that* $bet(B, C, D)$*.*
