High school geometry theorems

Hilbert's axiomatic system.
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Theorem 1 (th_4_01.) *Assuming that* $\alpha \neq \beta$ *and* $A \in \alpha$ *and* $A \in \beta$ *and* $p \in \alpha$ *and* $p \in \beta$ *there exist point* B*, point* C*, such that* $B \neq C$ *and* $B \in p$ *and* $C \in p$*.*

Proof:

1. There exist a point B and a point C where $B \neq C$ and $B \in p$ and $C \in p$ (using ax_I3a).
2. The conclusion follows from the facts $B \neq C$ and $B \in p$ and $C \in p$.

QED

Theorem 2 (th_4_02.) *Assuming that* $\alpha \neq \beta$ *and* $A \in \alpha$ *and* $A \in \beta$ *and* $p \in \alpha$ *and* $p \in \beta$ *and* $B \neq C$ *and* $B \in p$ *and* $C \in p$ *it holds that* $A \in p$*.*

Proof:

1. From the facts $p \in \alpha$ and $B \in p$ it holds that $B \in \alpha$ (using ax_D11).
2. From the facts $p \in \alpha$ and $C \in p$ it holds that $C \in \alpha$ (using ax_D11).
3. From the facts $p \in \beta$ and $B \in p$ it holds that $B \in \beta$ (using ax_D11).
4. From the facts $p \in \beta$ and $C \in p$ it holds that $C \in \beta$ (using ax_D11).
5. It holds that $A \in p$ or $A \notin p$.
   6. Assume that: $A \in p$.
      7. The conclusion follows from the fact $A \in p$.
   8. Assume that: $A \notin p$.
      9. From the facts $B \neq C$ and $B \in p$ and $C \in p$ and $A \notin p$ it holds that $\neg col(B, C, A)$ (using ax_D1a).
      10. From the facts $\neg col(B, C, A)$ and $B \in \alpha$ and $C \in \alpha$ and $A \in \alpha$ and $B \in \beta$ and $C \in \beta$ and $A \in \beta$ it holds that $\alpha = \beta$ (using ax_I5).
      11. From the facts $\alpha \neq \beta$ and $\alpha = \beta$ we get contradiction.
12. The conjecture follows in all cases.

QED
