

High school geometry theorems

Hilbert's axiomatic system.
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Theorem 1 (th_5.01.) *Assuming that $\neg col(A, B, C)$ it holds that $A \neq B$ and $B \neq C$ and $C \neq A$.*

Proof:

1. From the fact $\neg col(A, B, C)$ it holds that $\neg col(B, C, A)$ (using *ax_sym_ncol1*).
2. There exist a point D and a point E and a point F and a point G where $\neg cop(D, E, F, G)$ (using *ax_I8*).
3. It holds that $A = B$ or $A \neq B$.
4. Assume that: $A = B$.
5. It holds that $A = C$ or $A \neq C$.
6. Assume that: $A = C$.
7. From the facts $\neg col(A, B, C)$ and $A = B$ and $A = C$ it holds that $\neg col(A, A, A)$.
8. From the fact $\neg col(A, A, A)$ there exist a plane α , where $A \in \alpha$ and $A \in \alpha$ and $A \in \alpha$ (using *ax_I4a*).
9. It holds that $A = D$ or $A \neq D$.
10. Assume that: $A = D$.
11. It holds that $A = E$ or $A \neq E$.
12. Assume that: $A = E$.
13. It holds that $A = F$ or $A \neq F$.
14. Assume that: $A = F$.
15. From the facts $\neg col(A, B, C)$ and $A = B$ and $A = C$ and $A = D$ and $A = E$ and $A = F$ it holds that $\neg col(A, A, A)$.
16. From the facts $\neg cop(D, E, F, G)$ and $A = B$ and $A = C$ and $A = D$ and $A = E$ and $A = F$ it holds that $\neg cop(A, A, A, G)$.
17. From the facts $\neg col(A, A, A)$ and $A \in \alpha$ and $A \in \alpha$ and $A \in \alpha$ and $\neg cop(A, A, A, G)$ it holds that $G \notin \alpha$ (using *ax_D4a*).
18. It holds that $A = G$ or $A \neq G$.
19. Assume that: $A = G$.
20. From the facts $G \notin \alpha$ and $A = B$ and $A = C$ and $A = D$ and $A = E$ and $A = F$ and $A = G$ it holds that $A \notin \alpha$.
21. From the facts $A \notin \alpha$ and $A \in \alpha$ we get contradiction.
22. Assume that: $A \neq G$.
23. From the fact $A \neq G$ there exist a line p where $A \in p$ and $G \in p$ (using *ax_I1*).

24. From the facts $A \in p$ and $A \in p$ and $A \in p$ it holds that $col(A, A, A)$ (using ax_D1).
25. From the facts $\neg col(A, B, C)$ and $A = B$ and $A = C$ and $A = D$ and $A = E$ and $A = F$ it holds that $\neg col(A, A, A)$.
26. From the facts $\neg col(A, A, A)$ and $col(A, A, A)$ we get contradiction.
27. The conjecture follows in all cases.
28. Assume that: $A \neq F$.
29. From the fact $A \neq F$ there exist a line p where $A \in p$ and $F \in p$ (using ax_I1).
30. From the facts $A \in p$ and $A \in p$ and $A \in p$ it holds that $col(A, A, A)$ (using ax_D1).
31. From the facts $\neg col(A, B, C)$ and $A = B$ and $A = C$ and $A = D$ and $A = E$ it holds that $\neg col(A, A, A)$.
32. From the facts $\neg col(A, A, A)$ and $col(A, A, A)$ we get contradiction.
33. The conjecture follows in all cases.
34. Assume that: $A \neq E$.
35. From the fact $A \neq E$ there exist a line p where $A \in p$ and $E \in p$ (using ax_I1).
36. From the facts $A \in p$ and $A \in p$ and $A \in p$ it holds that $col(A, A, A)$ (using ax_D1).
37. From the facts $\neg col(A, B, C)$ and $A = B$ and $A = C$ and $A = D$ it holds that $\neg col(A, A, A)$.
38. From the facts $\neg col(A, A, A)$ and $col(A, A, A)$ we get contradiction.
39. The conjecture follows in all cases.
40. Assume that: $A \neq D$.
41. From the fact $A \neq D$ there exist a line p where $A \in p$ and $D \in p$ (using ax_I1).
42. From the facts $A \in p$ and $A \in p$ and $A \in p$ it holds that $col(A, A, A)$ (using ax_D1).
43. From the facts $\neg col(A, B, C)$ and $A = B$ and $A = C$ it holds that $\neg col(A, A, A)$.
44. From the facts $\neg col(A, A, A)$ and $col(A, A, A)$ we get contradiction.
45. The conjecture follows in all cases.
46. Assume that: $A \neq C$.
47. From the fact $A \neq C$ there exist a line p where $A \in p$ and $C \in p$ (using ax_I1).
48. From the facts $\neg col(B, C, A)$ and $A = B$ it holds that $\neg col(A, C, A)$.
49. From the facts $A \neq C$ and $A \in p$ and $C \in p$ and $\neg col(A, C, A)$ it holds that $A \notin p$ (using ax_D2a).
50. From the facts $A \notin p$ and $A \in p$ we get contradiction.
51. The conjecture follows in all cases.
52. Assume that: $A \neq B$.
53. From the fact $A \neq B$ there exist a line p where $A \in p$ and $B \in p$ (using ax_I1).
54. It holds that $A = C$ or $A \neq C$.
55. Assume that: $A = C$.
56. From the facts $\neg col(A, B, C)$ and $A = C$ it holds that $\neg col(A, B, A)$.

- 57. From the facts $A \neq B$ and $A \in p$ and $B \in p$ and $\neg col(A, B, A)$ it holds that $A \notin p$ (using ax_D2a).
 - 58. From the facts $A \notin p$ and $A \in p$ we get contradiction.
 - 59. Assume that: $A \neq C$.
 - 60. From the fact $A \neq C$ it holds that $C \neq A$.
 - 61. It holds that $B = C$ or $B \neq C$.
 - 62. Assume that: $B = C$.
 - 63. From the facts $\neg col(A, B, C)$ and $B = C$ it holds that $\neg col(A, B, B)$.
 - 64. From the facts $A \neq B$ and $A \in p$ and $B \in p$ and $\neg col(A, B, B)$ it holds that $B \notin p$ (using ax_D2a).
 - 65. From the facts $B \notin p$ and $B \in p$ we get contradiction.
 - 66. Assume that: $B \neq C$.
 - 67. The conclusion follows from the facts $A \neq B$ and $B \neq C$ and $C \neq A$.
 - 68. The conjecture follows in all cases.
 - 69. The conjecture follows in all cases.
 - 70. The conjecture follows in all cases.
- QED
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