High school geometry theorems

Hilbert's axiomatic system.
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Theorem 1 (th_6_01.) *Assuming that* $p \neq q$ *and* $q \neq r$ *and* $r \neq p$ *and lines* p *and* q *intersect and lines* q *and* r *intersect and lines* r *and* p *intersect and* $A \in p$ *and* $A \in q$ *and* $A \notin r$ *and* $B \notin p$ *and* $B \in q$ *and* $B \in r$ *and* $C \in p$ *and* $C \notin q$ *and* $C \in r$ *there exist plane* α*, such that* $A \in \alpha$ *and* $B \in \alpha$ *and* $C \in \alpha$*.*

Proof:

1. It holds that $A = B$ or $A \neq B$.
   2. Assume that: $A = B$.
      3. From the facts $B \notin p$ and $A = B$ it holds that $A \notin p$.
      4. From the facts $A \notin p$ and $A \in p$ we get contradiction.
   5. Assume that: $A \neq B$.
      6. From the facts $A \neq B$ and $A \in q$ and $B \in q$ and $C \notin q$ it holds that $\neg col(A, B, C)$ (using ax_D1a).
      7. From the fact $\neg col(A, B, C)$ there exist a plane α, where $A \in \alpha$ and $B \in \alpha$ and $C \in \alpha$ (using ax_I4a).
      8. The conclusion follows from the facts $A \in \alpha$ and $B \in \alpha$ and $C \in \alpha$.
9. The conjecture follows in all cases.

QED

Theorem 2 (th_6_02.) *Assuming that* $p \neq q$ *and* $q \neq r$ *and* $r \neq p$ *and lines* p *and* q *intersect and lines* q *and* r *intersect and lines* r *and* p *intersect and* $A \in p$ *and* $A \in q$ *and* $A \notin r$ *and* $B \notin p$ *and* $B \in q$ *and* $B \in r$ *and* $C \in p$ *and* $C \notin q$ *and* $C \in r$ *and* $A \in \alpha$ *and* $B \in \alpha$ *and* $C \in \alpha$ *it holds that* $p \in \alpha$ *and* $q \in \alpha$ *and* $r \in \alpha$*.*

Proof:

1. It holds that $A = B$ or $A \neq B$.
   2. Assume that: $A = B$.
      3. From the facts $B \notin p$ and $A = B$ it holds that $A \notin p$.
      4. From the facts $A \notin p$ and $A \in p$ we get contradiction.
   5. Assume that: $A \neq B$.
      6. From the facts $A \neq B$ and $A \in q$ and $B \in q$ and $A \in \alpha$ and $B \in \alpha$ it holds that $q \in \alpha$ (using ax_I6).
      7. It holds that $A = C$ or $A \neq C$.
         8. Assume that: $A = C$.
            9. From the facts $C \notin q$ and $A = C$ it holds that $A \notin q$.
            10. From the facts $A \notin q$ and $A \in q$ we get contradiction.
         11. Assume that: $A \neq C$.
            12. From the facts $A \neq C$ and $A \in p$ and $C \in p$ and $A \in \alpha$ and $C \in \alpha$ it holds that $p \in \alpha$ (using ax_I6).

14. Assume that: $B = C$.
   15. From the facts $C \in p$ and $B = C$ it holds that $B \in p$.
   16. From the facts $B \notin p$ and $B \in p$ we get contradiction.
17. Assume that: $B \neq C$.
   18. From the facts $B \neq C$ and $B \in r$ and $C \in r$ and $B \in \alpha$ and $C \in \alpha$ it holds that $r \in \alpha$ (using ax_I6).
   19. The conclusion follows from the facts $p \in \alpha$ and $q \in \alpha$ and $r \in \alpha$.
20. The conjecture follows in all cases.
21. The conjecture follows in all cases.
22. The conjecture follows in all cases.

QED