

High school geometry theorems

Hilbert's axiomatic system.
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Theorem 1 (th_2c_01.) *Assuming that $A \notin p$ and $A \in \alpha$ and $p \in \alpha$ and $A \in \beta$ and $p \in \beta$ there exist point B , point C , such that $B \neq C$ and $B \in p$ and $C \in p$.*

Proof:

1. There exist a point B and a point C where $B \neq C$ and $B \in p$ and $C \in p$ (using *ax_I3a*).
2. The conclusion follows from the facts $B \neq C$ and $B \in p$ and $C \in p$.

QED

Theorem 2 (th_2c_02.) *Assuming that $A \notin p$ and $A \in \alpha$ and $p \in \alpha$ and $A \in \beta$ and $p \in \beta$ and $B \neq C$ and $B \in p$ and $C \in p$ it holds that $col(A, B, C)$ or $\neg col(A, B, C)$.*

Proof:

1. It holds that $col(A, B, C)$ or $\neg col(A, B, C)$.
2. Assume that: $col(A, B, C)$.
3. The conclusion follows from the fact $col(A, B, C)$.
4. Assume that: $\neg col(A, B, C)$.
5. The conclusion follows from the fact $\neg col(A, B, C)$.
6. The conjecture follows in all cases.

QED

Theorem 3 (th_2c_03.) *Assuming that $A \in \alpha$ and $p \in \alpha$ and $A \in \beta$ and $p \in \beta$ and $B \neq C$ and $B \in p$ and $C \in p$ and $col(A, B, C)$ it holds that $A \in p$.*

Proof:

1. From the fact $col(A, B, C)$ it holds that $col(A, C, B)$ and $col(B, A, C)$ and $col(B, C, A)$ and $col(C, A, B)$ and $col(C, B, A)$ (using *ax_sym_col*).
2. It holds that $A \in p$ or $A \notin p$.
3. Assume that: $A \in p$.
4. The conclusion follows from the fact $A \in p$.
5. Assume that: $A \notin p$.
6. From the facts $B \neq C$ and $B \in p$ and $C \in p$ and $A \notin p$ it holds that $\neg col(B, C, A)$ (using *ax_D1a*).
7. From the facts $\neg col(B, C, A)$ and $col(B, C, A)$ we get contradiction.
8. The conjecture follows in all cases.

QED

Theorem 4 (th_2c_04.) *Assuming that $A \notin p$ and $A \in \alpha$ and $p \in \alpha$ and $A \in \beta$ and $p \in \beta$ and $B \neq C$ and $B \in p$ and $C \in p$ and $\neg \text{col}(A, B, C)$ it holds that $A \in \alpha$ and $B \in \alpha$ and $C \in \alpha$ and $A \in \beta$ and $B \in \beta$ and $C \in \beta$.*

Proof:

1. From the facts $p \in \alpha$ and $B \in p$ it holds that $B \in \alpha$ (using *ax_D11*).
2. From the facts $p \in \alpha$ and $C \in p$ it holds that $C \in \alpha$ (using *ax_D11*).
3. From the facts $p \in \beta$ and $B \in p$ it holds that $B \in \beta$ (using *ax_D11*).
4. From the facts $p \in \beta$ and $C \in p$ it holds that $C \in \beta$ (using *ax_D11*).
5. The conclusion follows from the facts $A \in \alpha$ and $B \in \alpha$ and $C \in \alpha$ and $A \in \beta$ and $B \in \beta$ and $C \in \beta$.

QED

Theorem 5 (th_2c_05.) *Assuming that $A \notin p$ and $A \in \alpha$ and $p \in \alpha$ and $A \in \beta$ and $p \in \beta$ and $B \neq C$ and $B \in p$ and $C \in p$ and $\neg \text{col}(A, B, C)$ and $A \in \alpha$ and $B \in \alpha$ and $C \in \alpha$ and $A \in \beta$ and $B \in \beta$ and $C \in \beta$ it holds that $\alpha = \beta$.*

Proof:

1. From the facts $\neg \text{col}(A, B, C)$ and $A \in \alpha$ and $B \in \alpha$ and $C \in \alpha$ and $A \in \beta$ and $B \in \beta$ and $C \in \beta$ it holds that $\alpha = \beta$ (using *ax_I5*).
2. The conclusion follows from the fact $\alpha = \beta$.

QED
