

High school geometry theorems

Hilbert's axiomatic system.
Formalized by: Sana Stojanovic Djurdjevic
Proved by: ArgoGeoChecker.

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Theorem 1 (th_5c2_01.) *Assuming that $A = B$ and $B = C$ it holds that $col(A, B, C)$.*

Proof:

1. There exist a point D and a point E and a point F where $\neg col(D, E, F)$ (using *ax_I3b*).
2. There exist a point G and a point I and a point J and a point K where $\neg cop(G, I, J, K)$ (using *ax_I8*).
3. It holds that $A = D$ or $A \neq D$.
4. Assume that: $A = D$.
5. It holds that $A = E$ or $A \neq E$.
6. Assume that: $A = E$.
7. It holds that $A = F$ or $A \neq F$.
8. Assume that: $A = F$.
9. From the facts $\neg col(D, E, F)$ and $A = B$ and $B = C$ and $A = D$ and $A = E$ and $A = F$ it holds that $\neg col(A, A, A)$.
10. From the fact $\neg col(A, A, A)$ there exist a plane α , where $A \in \alpha$ and $A \in \alpha$ and $A \in \alpha$ (using *ax_I4a*).
11. It holds that $A = G$ or $A \neq G$.
12. Assume that: $A = G$.
13. It holds that $A = I$ or $A \neq I$.
14. Assume that: $A = I$.
15. It holds that $A = J$ or $A \neq J$.
16. Assume that: $A = J$.
17. From the facts $\neg col(D, E, F)$ and $A = B$ and $B = C$ and $A = D$ and $A = E$ and $A = F$ and $A = G$ and $A = I$ and $A = J$ it holds that $\neg col(A, A, A)$.
18. From the facts $\neg cop(G, I, J, K)$ and $A = B$ and $B = C$ and $A = D$ and $A = E$ and $A = F$ and $A = G$ and $A = I$ and $A = J$ it holds that $\neg cop(A, A, A, K)$.
19. From the facts $\neg col(A, A, A)$ and $A \in \alpha$ and $A \in \alpha$ and $A \in \alpha$ and $\neg cop(A, A, A, K)$ it holds that $K \notin \alpha$ (using *ax_D4a*).
20. It holds that $A = K$ or $A \neq K$.
21. Assume that: $A = K$.

22. From the facts $K \notin \alpha$ and $A = B$ and $B = C$ and $A = D$ and $A = E$ and $A = F$ and $A = G$ and $A = I$ and $A = J$ and $A = K$ it holds that $A \notin \alpha$.
23. From the facts $A \notin \alpha$ and $A \in \alpha$ we get contradiction.
24. Assume that: $A \neq K$.
25. From the fact $A \neq K$ there exist a line p where $A \in p$ and $K \in p$ (using ax_I1).
26. From the facts $A \in p$ and $A \in p$ and $A \in p$ it holds that $col(A, A, A)$ (using ax_D1).
27. From the facts $\neg col(D, E, F)$ and $A = B$ and $B = C$ and $A = D$ and $A = E$ and $A = F$ and $A = G$ and $A = I$ and $A = J$ it holds that $\neg col(A, A, A)$.
28. From the facts $\neg col(A, A, A)$ and $col(A, A, A)$ we get contradiction.
29. The conjecture follows in all cases.
30. Assume that: $A \neq J$.
31. From the fact $A \neq J$ there exist a line p where $A \in p$ and $J \in p$ (using ax_I1).
32. From the facts $\neg col(D, E, F)$ and $A = B$ and $B = C$ and $A = D$ and $A = E$ and $A = F$ and $A = G$ and $A = I$ it holds that $\neg col(A, A, A)$.
33. From the facts $\neg col(A, A, A)$ and $col(A, A, A)$ we get contradiction.
34. The conjecture follows in all cases.
35. Assume that: $A \neq I$.
36. From the fact $A \neq I$ there exist a line p where $A \in p$ and $I \in p$ (using ax_I1).
37. From the facts $\neg col(D, E, F)$ and $A = B$ and $B = C$ and $A = D$ and $A = E$ and $A = F$ and $A = G$ it holds that $\neg col(A, A, A)$.
38. From the facts $\neg col(A, A, A)$ and $col(A, A, A)$ we get contradiction.
39. The conjecture follows in all cases.
40. Assume that: $A \neq G$.
41. From the fact $A \neq G$ there exist a line p where $A \in p$ and $G \in p$ (using ax_I1).
42. From the facts $\neg col(D, E, F)$ and $A = B$ and $B = C$ and $A = D$ and $A = E$ and $A = F$ it holds that $\neg col(A, A, A)$.
43. From the facts $\neg col(A, A, A)$ and $col(A, A, A)$ we get contradiction.
44. The conjecture follows in all cases.
45. Assume that: $A \neq F$.
46. From the fact $A \neq F$ there exist a line p where $A \in p$ and $F \in p$ (using ax_I1).
47. From the facts $col(A, A, A)$ and $A = B$ and $A = C$ it holds that $col(A, B, C)$.

48. The conclusion follows from the fact $col(A, B, C)$.
49. The conjecture follows in all cases.
50. Assume that: $A \neq E$.
51. From the fact $A \neq E$ there exist a line p where $A \in p$ and $E \in p$ (using ax_I1).
52. From the facts $col(A, A, A)$ and $A = B$ and $A = C$ it holds that $col(A, B, C)$.
53. The conclusion follows from the fact $col(A, B, C)$.
54. The conjecture follows in all cases.
55. Assume that: $A \neq D$.
56. From the fact $A \neq D$ there exist a line p where $A \in p$ and $D \in p$ (using ax_I1).
57. From the facts $col(A, A, A)$ and $A = B$ and $A = C$ it holds that $col(A, B, C)$.
58. The conclusion follows from the fact $col(A, B, C)$.
59. The conjecture follows in all cases.

QED

Theorem 2 (th_5c2_02.) *Assuming that $\neg col(A, B, C)$ and $A = B$ and $B \neq C$ there exist line p , such that $B \in p$ and $C \in p$.*

Proof:

1. From the facts $B \neq C$ and $A = B$ it holds that $A \neq C$.
2. From the fact $A \neq C$ there exist a line p where $A \in p$ and $C \in p$ (using ax_I1).
3. From the facts $A \in p$ and $A = B$ it holds that $B \in p$.
4. The conclusion follows from the facts $B \in p$ and $C \in p$.

QED

Theorem 3 (th_5c2_03.) *Assuming that $A = B$ and $B \neq C$ and $B \in p$ and $C \in p$ it holds that $col(A, B, C)$.*

Proof:

1. From the facts $B \in p$ and $A = B$ it holds that $A \in p$.
2. From the facts $A \in p$ and $A \in p$ and $C \in p$ it holds that $col(A, A, C)$ (using ax_D1).
3. From the facts $col(A, A, C)$ and $A = B$ it holds that $col(A, B, C)$.
4. The conclusion follows from the fact $col(A, B, C)$.

QED

Theorem 4 (th_5c2_04.) *Assuming that $\neg col(A, B, C)$ and $A \neq B$ it holds that $B \neq C$ and $C \neq A$.*

Proof:

1. From the fact $A \neq B$ there exist a line p where $A \in p$ and $B \in p$ (using ax_I1).
2. It holds that $A = C$ or $A \neq C$.
3. Assume that: $A = C$.
4. From the facts $\neg col(A, B, C)$ and $A = C$ it holds that $\neg col(A, B, A)$.

5. From the facts $A \neq B$ and $A \in p$ and $B \in p$ and $\neg col(A, B, A)$ it holds that $A \notin p$ (using *ax_D2a*).
6. From the facts $A \notin p$ and $A \in p$ we get contradiction.
7. Assume that: $A \neq C$.
8. From the fact $A \neq C$ it holds that $C \neq A$.
9. It holds that $B = C$ or $B \neq C$.
10. Assume that: $B = C$.
11. From the facts $\neg col(A, B, C)$ and $B = C$ it holds that $\neg col(A, B, B)$.
12. From the facts $A \neq B$ and $A \in p$ and $B \in p$ and $\neg col(A, B, B)$ it holds that $B \notin p$ (using *ax_D2a*).
13. From the facts $B \notin p$ and $B \in p$ we get contradiction.
14. Assume that: $B \neq C$.
15. The conclusion follows from the facts $B \neq C$ and $C \neq A$.
16. The conjecture follows in all cases.
17. The conjecture follows in all cases.

QED
