

High school geometry theorems

Hilbert's axiomatic system.
Formalized by: Sana Stojanovic Djurdjevic
Proved by: ArgoGeoChecker.

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Theorem 1 (th_6c_01.) *Assuming that $p \neq q$ and $q \neq r$ and $r \neq p$ and lines p and q intersect and lines q and r intersect and lines r and p intersect and $A \in p$ and $A \in q$ and $A \notin r$ and $B \notin p$ and $B \in q$ and $B \in r$ and $C \in p$ and $C \notin q$ and $C \in r$ it holds that $A \neq B$ and $B \neq C$ and $C \neq A$.*

Proof:

1. It holds that $A = B$ or $A \neq B$.
2. Assume that: $A = B$.
3. From the facts $B \notin p$ and $A = B$ it holds that $A \notin p$.
4. From the facts $A \notin p$ and $A \in p$ we get contradiction.
5. Assume that: $A \neq B$.
6. It holds that $A = C$ or $A \neq C$.
7. Assume that: $A = C$.
8. From the facts $C \notin q$ and $A = C$ it holds that $A \notin q$.
9. From the facts $A \notin q$ and $A \in q$ we get contradiction.
10. Assume that: $A \neq C$.
11. From the fact $A \neq C$ it holds that $C \neq A$.
12. It holds that $B = C$ or $B \neq C$.
13. Assume that: $B = C$.
14. From the facts $C \in p$ and $B = C$ it holds that $B \in p$.
15. From the facts $B \notin p$ and $B \in p$ we get contradiction.
16. Assume that: $B \neq C$.
17. The conclusion follows from the facts $A \neq B$ and $B \neq C$ and $C \neq A$.
18. The conjecture follows in all cases.
19. The conjecture follows in all cases.
20. The conjecture follows in all cases.

QED

Theorem 2 (th_6c_02.) *Assuming that $p \neq q$ and $q \neq r$ and $r \neq p$ and lines p and q intersect and lines q and r intersect and lines r and p intersect and $A \in p$ and $A \in q$ and $A \notin r$ and $B \notin p$ and $B \in q$ and $B \in r$ and $C \in p$ and $C \notin q$ and $C \in r$ and $A \neq B$ and $B \neq C$ and $C \neq A$ it holds that $col(A, B, C)$ or $\neg col(A, B, C)$.*

Proof:

1. It holds that $col(A, B, C)$ or $\neg col(A, B, C)$.
2. Assume that: $col(A, B, C)$.
3. The conclusion follows from the fact $col(A, B, C)$.
4. Assume that: $\neg col(A, B, C)$.
5. The conclusion follows from the fact $\neg col(A, B, C)$.
6. The conjecture follows in all cases.

QED

Theorem 3 (th_6c_03.) *Assuming that $p \neq q$ and $q \neq r$ and $r \neq p$ and lines p and q intersect and lines q and r intersect and lines r and p intersect and $A \in p$ and $A \in q$ and $A \notin r$ and $B \notin p$ and $B \in q$ and $B \in r$ and $C \in p$ and $C \notin q$ and $C \in r$ and $A \neq B$ and $B \neq C$ and $C \neq A$ and $col(A, B, C)$ there exist line s , such that $A \in s$ and $B \in s$ and $C \in s$.*

Proof:

1. From the fact $col(A, B, C)$ there exist a line s where $A \in s$ and $B \in s$ and $C \in s$ (using $ax.D2$).
2. The conclusion follows from the facts $A \in s$ and $B \in s$ and $C \in s$.

QED

Theorem 4 (th_6c_04.) *Assuming that $p \neq q$ and $q \neq r$ and $r \neq p$ and lines p and q intersect and lines q and r intersect and lines r and p intersect and $A \in p$ and $A \in q$ and $A \notin r$ and $B \notin p$ and $B \in q$ and $B \in r$ and $C \in p$ and $C \notin q$ and $C \in r$ and $A \neq B$ and $B \neq C$ and $C \neq A$ and $col(A, B, C)$ and $A \in s$ and $B \in s$ and $C \in s$ it holds that $s = p$ and $s = q$ and $s = r$.*

Proof:

1. From the facts $A \neq B$ and $A \in q$ and $B \in q$ and $A \in s$ and $B \in s$ it holds that $q = s$ (using $ax.I2$).
2. From the facts $C \in s$ and $q = s$ it holds that $C \in q$.
3. From the facts $C \notin q$ and $C \in q$ we get contradiction.

QED

Theorem 5 (th_6c_05.) *Assuming that $p \neq q$ and $q \neq r$ and $r \neq p$ and lines p and q intersect and lines q and r intersect and lines r and p intersect and $A \in p$ and $A \in q$ and $A \notin r$ and $B \notin p$ and $B \in q$ and $B \in r$ and $C \in p$ and $C \notin q$ and $C \in r$ and $A \neq B$ and $B \neq C$ and $C \neq A$ and $\neg col(A, B, C)$ there exist plane α , such that $A \in \alpha$ and $B \in \alpha$ and $C \in \alpha$.*

Proof:

1. From the fact $\neg col(A, B, C)$ there exist a plane α , where $A \in \alpha$ and $B \in \alpha$ and $C \in \alpha$ (using $ax.I4a$).
2. The conclusion follows from the facts $A \in \alpha$ and $B \in \alpha$ and $C \in \alpha$.

QED

Theorem 6 (th_6c_06.) *Assuming that $p \neq q$ and $q \neq r$ and $r \neq p$ and lines p and q intersect and lines q and r intersect and lines r and p intersect and $A \in p$ and $A \in q$ and $A \notin r$ and $B \notin p$ and $B \in q$ and $B \in r$ and $C \in p$ and $C \notin q$ and $C \in r$ and $A \neq B$ and $B \neq C$ and $C \neq A$ and $\neg col(A, B, C)$ and $A \in \alpha$ and $B \in \alpha$ and $C \in \alpha$ it holds that $p \in \alpha$ and $q \in \alpha$ and $r \in \alpha$.*

Proof:

1. From the facts $A \neq B$ and $A \in q$ and $B \in q$ and $A \in \alpha$ and $B \in \alpha$ it holds that $q \in \alpha$ (using *ax_I6*).
2. From the fact $C \neq A$ it holds that $A \neq C$.
3. From the facts $A \neq C$ and $A \in p$ and $C \in p$ and $A \in \alpha$ and $C \in \alpha$ it holds that $p \in \alpha$ (using *ax_I6*).
4. From the facts $B \neq C$ and $B \in r$ and $C \in r$ and $B \in \alpha$ and $C \in \alpha$ it holds that $r \in \alpha$ (using *ax_I6*).
5. The conclusion follows from the facts $p \in \alpha$ and $q \in \alpha$ and $r \in \alpha$.

QED
