

High school geometry theorems

Hilbert's axiomatic system.
Formalized by: Sana Stojanovic Djurdjevic
Proved by: ArgoGeoChecker.

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Theorem 1 (th_7_01.) *Assuming that lines p and q intersect there exist point A , such that $A \in p$ and $A \in q$.*

Proof:

1. From the fact lines p and q intersect there exist a point A where $p \neq q$ and $A \in p$ and $A \in q$ (using *ax_D6*).
2. The conclusion follows from the facts $A \in p$ and $A \in q$.

QED

Theorem 2 (th_7_02.) *Assuming that lines p and q intersect and $A \in p$ and $A \in q$ there exist point B , such that $B \neq A$ and $B \in p$.*

Proof:

1. There exist a point B and a point C where $B \neq C$ and $B \in p$ and $C \in p$ (using *ax_I3a*).
2. It holds that $A = B$ or $A \neq B$.
3. Assume that: $A = B$.
 4. From the facts $B \neq C$ and $A = B$ it holds that $A \neq C$.
 5. From the facts $A \neq C$ and $C = A$ and $A = C$ it holds that $C \neq A$.
 6. From the fact $A \neq C$ it holds that $C \neq A$.
 7. The conclusion follows from the facts $C \neq A$ and $C \in p$.
8. Assume that: $A \neq B$.
 9. From the fact $A \neq B$ it holds that $B \neq A$.
 10. The conclusion follows from the facts $B \neq A$ and $B \in p$.
11. The conjecture follows in all cases.

QED

Theorem 3 (th_7_03.) *Assuming that lines p and q intersect and $A \in p$ and $A \in q$ and $B \neq A$ and $B \in p$ there exist point C , such that $C \neq A$ and $C \in q$.*

Proof:

1. There exist a point C and a point D where $C \neq D$ and $C \in q$ and $D \in q$ (using *ax_I3a*).
2. It holds that $A = C$ or $A \neq C$.
3. Assume that: $A = C$.
 4. From the facts $C \neq D$ and $A = C$ it holds that $A \neq D$.
 5. From the facts $A \neq D$ and $D = A$ and $A = D$ it holds that $D \neq A$.

6. From the fact $A \neq D$ it holds that $D \neq A$.
7. The conclusion follows from the facts $D \neq A$ and $D \in q$.
8. Assume that: $A \neq C$.
9. From the fact $A \neq C$ it holds that $C \neq A$.
10. The conclusion follows from the facts $C \neq A$ and $C \in q$.
11. The conjecture follows in all cases.

QED

Theorem 4 (th_7_04.) *Assuming that lines p and q intersect and $A \in p$ and $A \in q$ and $B \neq A$ and $B \in p$ and $C \neq A$ and $C \in q$ it holds that $\neg col(A, B, C)$.*

Proof:

1. From the fact $B \neq A$ it holds that $A \neq B$.
2. From the fact $C \neq A$ it holds that $A \neq C$.
3. It holds that $B \in q$ or $B \notin q$.
4. Assume that: $B \in q$.
5. From the facts $A \neq B$ and $A \in p$ and $B \in p$ and $A \in q$ and $B \in q$ it holds that $p = q$ (using *ax_I2*).
6. From the facts lines p and q intersect and $p = q$ it holds that lines p and p intersect.
7. From the fact lines p and p intersect there exist a point D where $p \neq p$ and $D \in p$ and $D \in p$ (using *ax_D6*).
8. From the fact $p \neq p$ we get contradiction.
9. Assume that: $B \notin q$.
10. It holds that $C \in p$ or $C \notin p$.
11. Assume that: $C \in p$.
12. From the facts $A \neq C$ and $A \in p$ and $C \in p$ and $A \in q$ and $C \in q$ it holds that $p = q$ (using *ax_I2*).
13. From the facts $B \notin q$ and $p = q$ it holds that $B \notin p$.
14. From the facts $B \notin q$ and $p = q$ it holds that $B \notin p$.
15. From the facts lines p and q intersect and $p = q$ it holds that lines p and p intersect.
16. From the fact $p \neq p$ we get contradiction.
17. Assume that: $C \notin p$.
18. From the facts $A \neq B$ and $A \in p$ and $B \in p$ and $C \notin p$ it holds that $\neg col(A, B, C)$ (using *ax_D1a*).
19. The conclusion follows from the fact $\neg col(A, B, C)$.
20. The conjecture follows in all cases.
21. The conjecture follows in all cases.

QED

Theorem 5 (th_7_05.) *Assuming that lines p and q intersect and $A \in p$ and $A \in q$ and $B \neq A$ and $B \in p$ and $C \neq A$ and $C \in q$ and $\neg col(A, B, C)$ there exist plane α , such that $A \in \alpha$ and $B \in \alpha$ and $C \in \alpha$.*

Proof:

1. From the fact $\neg col(A, B, C)$ there exist a plane α , where $A \in \alpha$ and $B \in \alpha$ and $C \in \alpha$ (using *ax_I4a*).
2. The conclusion follows from the facts $A \in \alpha$ and $B \in \alpha$ and $C \in \alpha$.

QED

Theorem 6 (th_7_06.) *Assuming that lines p and q intersect and $A \in p$ and $A \in q$ and $B \neq A$ and $B \in p$ and $C \neq A$ and $C \in q$ and $\neg \text{col}(A, B, C)$ and $A \in \alpha$ and $B \in \alpha$ and $C \in \alpha$ it holds that $p \in \alpha$ and $q \in \alpha$.*

Proof:

1. From the fact $B \neq A$ it holds that $A \neq B$.
2. From the facts $A \neq B$ and $A \in p$ and $B \in p$ and $A \in \alpha$ and $B \in \alpha$ it holds that $p \in \alpha$ (using *ax_I6*).
3. From the fact $C \neq A$ it holds that $A \neq C$.
4. From the facts $A \neq C$ and $A \in q$ and $C \in q$ and $A \in \alpha$ and $C \in \alpha$ it holds that $q \in \alpha$ (using *ax_I6*).
5. The conclusion follows from the facts $p \in \alpha$ and $q \in \alpha$.

QED
