# Instance-Based Selection of Policies for SAT Solvers

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# Overview

1. Problem Description
2. Methodology
3. Experiments and Evaluation
   - Training Phase
   - Analysis
   - Exploitation
4. Related Work
5. Conclusions and Further Work

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# SAT Solvers and Their Policies

- DPLL-based, conflict driven SAT solvers are considered
- Heuristic policies (variable selection, restarts, forgetting. . . ) control SAT solver behaviour
- Strategies are fixed combinations of policies
- Formulae from the applications are typically clustered in the families

# SAT Solvers and Their Policies

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- Heuristic policies (variable selection, restarts, forgetting. . . ) control SAT solver behaviour
- Strategies are fixed combinations of policies
- Formulae from the applications are typically clustered in the families
- Main idea: find a family of formulae most similar to the formula being solved, and use the strategy that has performed good for formulae of that family?

# Hypotheses

(H1) Formulae of the same family share some syntactical properties that can be used for automated formula classification

(H2) For each family of formulae there is only a small number of solving strategies that are appropriate — that show better performance on formulae belonging to that family then all other available strategies

(H3) For syntactically similar formulae, the best strategies are also similar in some sense

## Goals

- Analyze a relationship between the syntax of a formulae and their appropriate solving strategies
- Develop a methodology for intelligent choosing of solvers' strategies, based on the syntax of the input formula, and use it to improve the efficiency of a SAT solver

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# Overview of the Methodology

1. Training
   - Solving formulae from a large corpus for all given strategies
   - Determining which strategies are best for different families of formulae
2. Analysis
   - Verification of the hypotheses (H1-H3)
3. Exploitation
   - Classification of an unknown input formula to some of the families
   - Solving the input formula using the strategy best for the chosen family

# Corpora of Formulae

- Corpus SAT 2002
  - 1964 formulae
  - 39 families
- Corpus SAT 2007
  - 906 formulae

# SAT Solver and Strategies Used

SAT solver: ArgoSAT

**Variable selection**

$\mathrm{VS}_{\mathit{random}}$, $\mathrm{VS}_{\mathit{VSIDS}}^{1.0,\,1.0/0.95,\,\mathit{freq}}$, $\mathrm{VS}_{\mathit{random}}^{0.05} \circ \mathrm{VS}_{\mathit{VSIDS}}^{1.0,\,1.0/0.95,\,\mathit{freq}}$

**Polarity selection**

$\mathrm{PS}_{\mathit{pos}}$, $\mathrm{PS}_{\mathit{neg}}$, $\mathrm{PS}_{\mathit{random}}^{0.5}$, $\mathrm{PS}_{\mathit{polarity\_caching}}^{\mathit{neg}}$, $\mathrm{PS}_{\mathit{polarity\_caching}}^{\mathit{freq}}$

**Restart**

$\mathrm{RS}_{\mathit{no\_restart}}$, $\mathrm{RS}_{\mathit{minisat}}^{100,1.5}$, $\mathrm{RS}_{\mathit{luby}}^{512}$, $\mathrm{RS}_{\mathit{picosat}}^{100,1.5}$

# Classification of Propositional Formulae

- Represent formulae by vectors of numerical features
- Perform classification over such vectors using standard classification techniques

# Choice of Features

Formulae are represented by 33 *syntactical* features:

- The number of clauses $c$ and variables $v$ in the input formula, and their ratio $\frac{c}{v}$,
- Node degree statistics for variable nodes in variable-clause graph: mean, variation coefficient, minimum, maximum and entropy,
- Fraction of binary clauses, ternary clauses, and Horn clauses,
- ...

# Classification Method

Formulae are classified by *k* Nearest Neighbours method with the following distance function

$$d(f_1, f_2) = \sum_{x \in features} \frac{|f_1(x) - f_2(x)|}{\sqrt{|f_1(x) f_2(x)|} + 1}$$

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# Training Details

- Cutoff time for solving used is 10 minutes
- Clause and variable shuffling
- Total number of SAT solver calls:
  1964 formulae × 2 shuffled variants × 60 strategies $=$ 235.680
- A cluster computer with 32 processors was used
- Total computing time on the cluster was around a month.

# Evaluation of Formulae Classification (H1)

*(H1) Formulae can be automatically classified into families based on their syntax*

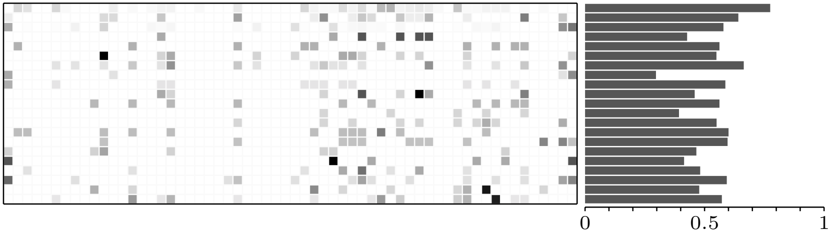
- Corpus SAT 2002
- Average feature computation and classification time for a formula was 0.4s
- The best precision was achieved by 1 Nearest Neighbour
- The leave one out procedure was used
- Precision was 92.5%

# Analysis of Appropriate Strategies (H2)

*(H2) For each family of formulae there is only a small number of appropriate solving strategies*

- For each family and each strategy the percentage of formulae for which that strategy is better than the others was considered
- The normalized entropies for families were calculated

# Analysis of Appropriate Strategies (H2)



# Similarity of Formulae and Their Best Strategies (H3)

*(H3) For syntactically similar formulae, the best strategies are also similar*

- For each 2 formulae form the corpus do:
  - Calculate distance between them
  - Calculate edit distance between their best strategies
- The Pearson correlation coefficient between these distances is 0.51

# Strategy Selection Procedures

Best fixed - for each formula use the same strategy that is the best on the training corpus

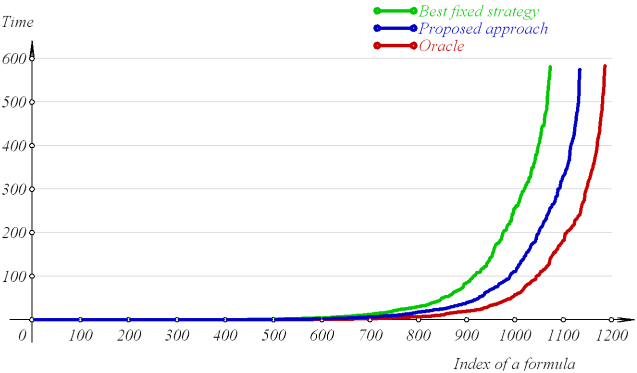
Proposed - classify the formula and use the strategy best for the chosen family

Oracle - for each formula use the strategy that is the best for it

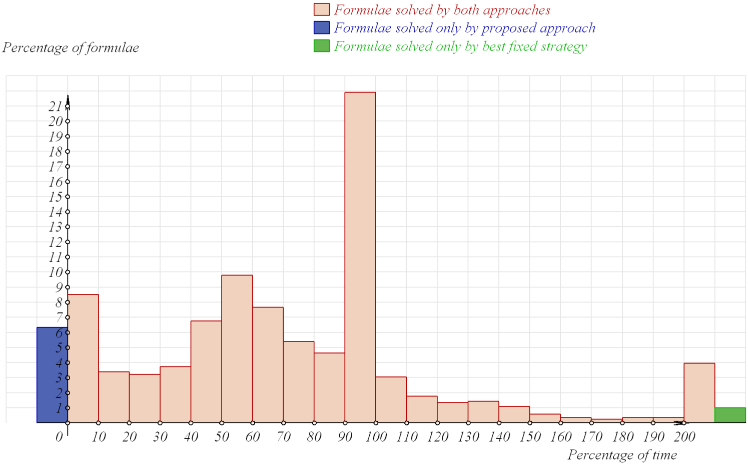
# Evaluation of Strategy Selection Procedures on SAT2002

| Procedure | No. solved | Median time |
|---|---|---|
| Best fixed | 1073 | 207.4s |
| Proposed | 1135 | 92.6s |
| Oracle | 1187 | 45.8 |

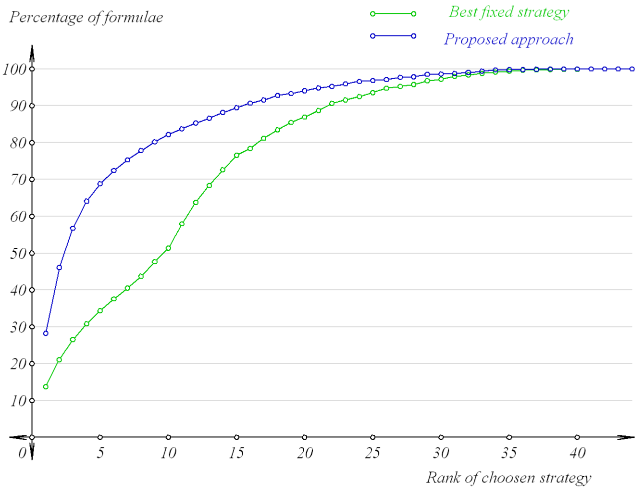
# Solving Times for Different Strategy Selection Procedures



# Speedup Histogram for the Proposed Procedure



# CDF of Ranks of Chosen Strategies



# Evaluation on SAT2007.

ArgoSmArt = ArgoSAT + proposed strategy selection procedure

| System | No. solved | 20-th perc. time |
|---|---|---|
| ArgoSAT | 219 | 311.6s |
| ArgoSmArt | 239 | 249.5s |

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# Related Work

- L. Xu, F. Hutter, H. Hoos, K. Leyton-Brown. SATzilla: Portfolio-Based Algorithm Selection for SAT. *Journal of Artificial Intelligence Research*, 2008.
- S. Haim, T. Walsh. Online Estimation of SAT Solving Runtime. *SAT 2008*.
- H. Samulowitz, R. Memisevic. Learning to Solve QBF. *AAAI 2007*.
- ...

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# Conclusions and Further Work

- Intelligent choosing of solvers' strategies, based on the syntax of the input formula, can significantly improve efficiency of a SAT solver
- Further work:
  - Further analysis of gathered data
  - Stochastic parameter optimization
  - On-line control of SAT solver