The Tool GCLC and Links Between Automated Deduction and Dynamic Geometry

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AD + GeoGebra
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Agenda

- Brief survey of automated theorem proving in geometry
- The Tool GCLC/WinGCLC
- Links Between Automated Deduction and Dynamic Geometry

Geometrical Theorems of Constructive Type

- Usually, only Euclidean plane geometry
- Conjectures that corresponds to properties of constructions
- Non-degenerate conditions are very important

Automated Theorem Proving in Geometry

- Around for more than 50 years
- Early AI-based approaches in 50's
- Huge successes made by algebraic theorem provers in 80's
- Successes made by coordinate-free methods in 90's
- Synthetic proofs generated by provers using coherent logic in 2000's

General Setting

- No synthetic geometry proofs, only algebraic justification
- Premises and goals have the form of equalities
- Construction steps are converted into a polynomial system

$$\begin{array}{rcl} h_1(u_1, u_2, \ldots, u_d, x_1, \ldots, x_n) & = & 0 \\ \ldots & & \\ h_t(u_1, u_2, \ldots, u_d, x_1, \ldots, x_n) & = & 0 \end{array}$$

- Check whether for the conjecture it holds that

$$g(u_1, u_2, \ldots, u_d, x_1, \ldots, x_n) = 0$$

Two Most Significant Methods:

Very efficient, but do not produce readable proofs:

- Gröbner-bases Method: invented by Buchberger in 1965, widely used algorithm with many applications
- Wu's Method: invented by Wu in 1977 (often considered to be one of the most efficient provers overall)

Most Significant Methods:

Attempt to give traditional (human readable) proofs:

- Area method (Chou et.al.1990's)
- Full-angle method (Chou et.al.1990's)
- Vector Method (Chou et.al.1990's)

Theorem Provers based on Coherent Logic (Geometry Logic)

- Only formulae of the following form are considered:

$$\forall \vec{x}(A_1 \wedge \ldots \wedge A_n) \Rightarrow \exists \vec{y}(B_1 \vee \ldots \vee B_m))$$

- Produces fully readable, synthetic geometry proofs
- Suitable for foundational conjectures (close to the level of axioms)
- Automated by Kordić and Janičić (1993), Bezem et all (2000's)

Some of Existing Implementations

- GEX family (Chou et al): Wu's, GB, area, vector, full-angle method
- GeoProof (Narboux): area, GB
- Theorema (Robu et al): area, GB
- Geometry Explorer (Wilson et al): full-angle
- GCLC (Janičić/Quaresma/Predović): area, Wu's, GB
- Discover: GB
- ...

GCLC/WinGCLC

- Developed since 1996.
- Originally, a tool for producing geometrical illustrations in LaTeX, today — much more than that
- Freely available from
  http://www.matf.bg.ac.rs/~janicic/gclc
- Used in:
  - producing digital mathematical illustrations
  - mathematical education
  - storing mathematical contents
  - studies of automated geometrical reasoning

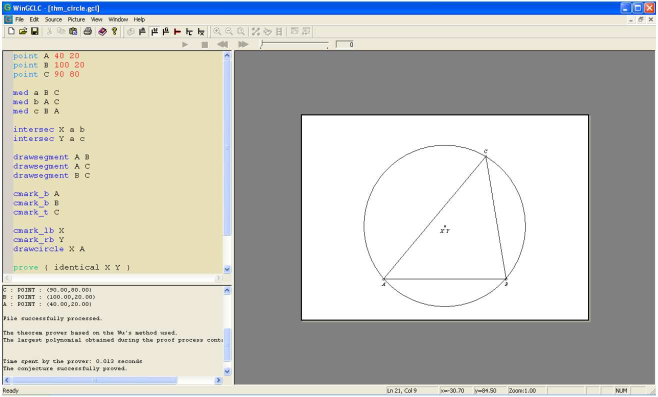
Distinctive Features

- It is not based on point-and-click approach, but on the language for specifying both contents and presentation
- There are three theorem provers tightly integrated in GCLC:
  - a theorem prover based on Wu's method
  - a theorem prover based on GB method
  - a theorem prover based on the area method

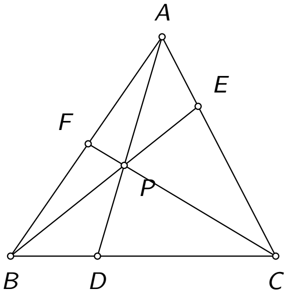
Theorem Provers Built-into GCLC

- All of them are very efficient and can prove many non-trivial theorems in only milliseconds
- The provers are tightly built-in: the user has just to state the conjecture after the description of a construction, for example:
  `prove { identical O_1 O_2 }`

Short Demo of WinGCLC



Example: Ceva's Theorem



- Conjecture:

$$\frac{\overrightarrow{AF}}{\overrightarrow{FB}} \cdot \frac{\overrightarrow{BD}}{\overrightarrow{DC}} \cdot \frac{\overrightarrow{CE}}{\overrightarrow{EA}} = 1$$

- Outputs by the three provers...

Purposes:

- Educational purposes
- Proof visualizations (jGeX)
- Verifying construction steps (GCLC, Cinderella)

Links Between AD and DG: Technical Level (1)

- Implementing a new prover from a scratch is demanding
- Good to avoid duplicating efforts
- Good to define standard communication

Links Between AD and DG: Technical Level (2)

Via API?

```
CTheoremProver *pProver = new CAreaMethod();
pProver->AddProverCommand(p_point, "A", "20", "20");
pProver->AddProverCommand(p_point, "B", "50", "20");
pProver->AddProverCommand(p_point, "C", "40", "70");
pProver->AddProverCommand(midpoint, "B1", "B", "C");
pProver->AddProverCommand(midpoint, "A1", "A", "C");
pProver->AddProverConjecture("equal { sratio A B A1 B1 } { 2
}");
r = pProver->Prove(hProofLaTeXOutput, hProofXMLOutput, &Time,
theorem, &e);
```

Links Between AD and DG: Technical Level (3)

Via XML files? [Related to I2geo format?]

```
<figure>
   <define>
      <fixed_point x="20.000000" y="10.000000">A</fixed_point>
      <fixed_point x="70.000000" y="10.000000">B</fixed_point>
      <fixed_point x="35.000000" y="40.000000">C</fixed_point>
   </define>
   <construct>
      <midpoint> <new_point>B_1</new_point>
         <point>B</point> <point>C</point> </midpoint>
      <midpoint> <new_point>A_1</new_point>
         <point>A</point> <point>C</point> </midpoint>
   </construct>
...
</figure>
```

Issues

- Where the provers would be the most helpful in dynamic geometry?
- Define a standard communication between provers and GeoGebra/dynamic tools
- Set up a database of conjectures (something like GeoThms)?